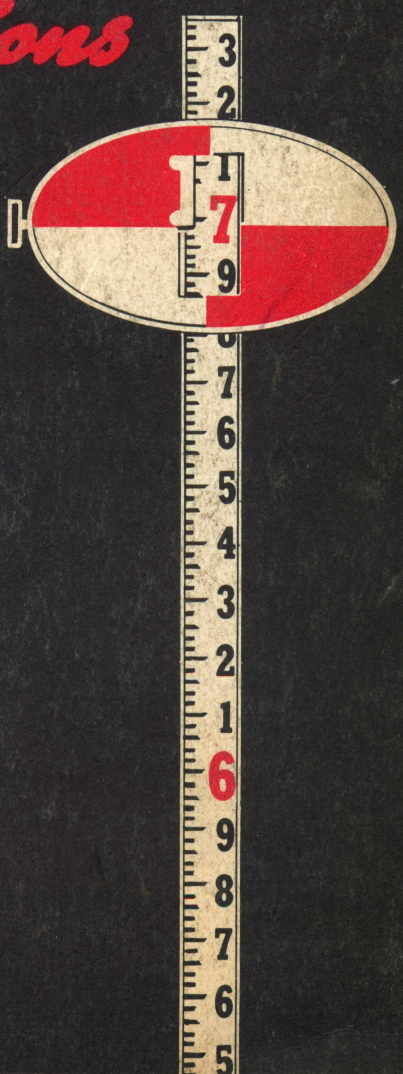
# MONROE

## *Machine Methods for*

# CIVIL ENGINEERING

0.52

## *Calculations*



# MONROE

## MACHINE METHODS FOR

## CIVIL ENGINEERING

## CALCULATIONS

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## MACHINE METHODS FOR

# CIVIL ENGINEERING

## CALCULATIONS

with instructions for

MONROE CALCULATOR 8N

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## AN OPENING NOTE

Because civil engineering is concerned with figuring to a much greater degree than any other profession, the appearance of this booklet needs no justification.

Like several editions of a previous Monroe publication on the same subject, the purpose is to provide engineers and contractors with practical information for applying the modern calculator to the figuring problems they most frequently encounter in their daily work and to suggest time and labor saving short-cuts. The Monroe machine methods have been updated, several incorporating the latest, improved techniques, and new material has been added. Furthermore, for the benefit of those who have had little or no experience in operating a Monroe Calculator, there is an introductory section with detailed instructions for the fully automatic 8N which is the model most generally found in engineering offices. While the engineering applications that make up Part II are also for the 8N Model, they are described in such a way that users of other type Monroes can easily adapt the procedures to their machines.

Although far from being a textbook on engineering mathematics, this handbook offers Monroe machine methods for a large part of the more common kinds of calculations required of the civil engineer. Anyone with questions or needing help in applying the Monroe to other figuring jobs is invited to call upon the services of any branch or local office of the Monroe Calculating Machine Company, Inc. Assistance and personal instructions are always gladly given without cost or obligation.

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# MONROE MACHINE METHODS

# CIVIL ENGINEERING CALCULATIONS

## PART I

## MONROE OPERATING INSTRUCTIONS

### AN INTRODUCTION

*for the engineer just starting to use the Monroe*

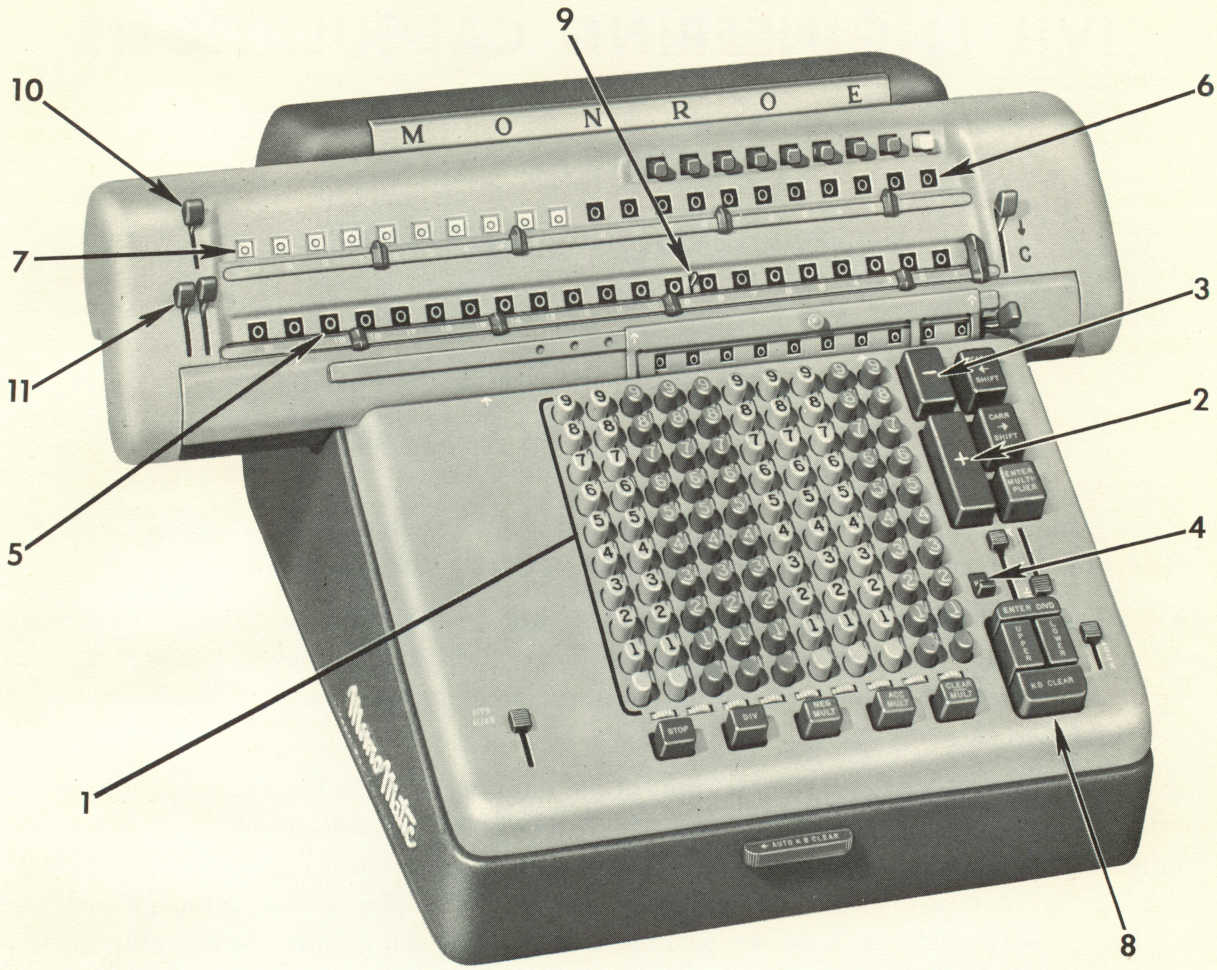
You are going to like the Monroe and very soon it will be your most indispensable tool, just as it is with thousands of engineers whose daily work calls for so much figuring.

The Monroe is noted for its simplicity and you will be amazed how quickly you become proficient in operating it and applying it to all your jobs. First, however, you will want to familiarize yourself with the machine and learn the recommended ways of using it in order to get the fullest benefit from it.

The Monroe is an all-round figuring machine of the rotary type that adds, subtracts, multiplies, and divides, also extracts roots. As it is extremely flexible, it handles every kind of figuring from the most ordinary to the most complicated, in a straightforward way with a minimum of presetting. All Monroe models have essentially the same principles so that, although each one has certain special features, a person who has operated one can readily use any other.

Part I of this manual gives detailed information about the fully automatic Monroe Model 8N which is probably the most popular machine with engineers because of its efficiency and time-saving on their specialized computations. The first few pages describe the keys and operating controls of the 8N and explain their purpose and how they are used. These are followed by step-by-step instructions for performing the basic arithmetical processes and a few elementary problems. Naturally it is highly desirable that the beginner study the first section rather carefully before taking up the engineering applications of Part II.

## MONROE CALCULATOR MONRO-MATIC MODEL 8N




1 Keyboard
2 Plus Bar
3 Minus Bar
4 Repeat and Non-repeat Lever
5 Lower Dials
6 Right Upper Dials
7 Left Upper Dials
8 Clear Keys
9 Half-cent Control
10 Upper Dials Lock
11 Lower Dials Locks

## THE OPERATING CONTROLS

All the principal operating levers and keys are marked to indicate the functions they control. As a further aid to the operator, controls relating to multiplication are colored green and those for division, coral.

**1 Keyboard** Every Monroe Calculator has a single keyboard by which all figures are entered for all operations. The unnumbered key at the bottom of each column is the so-called "zero key." When one of these is depressed it will clear whatever numeral key may be depressed in the same column. Depressing the zero key with one finger while depressing simultaneously a numeral key in the same column with another finger locks that figure against normal clearance. The locked figure is released by depressing another numeral key in the same column.

**2 Plus Bar** The plus bar is used for addition. Each time it is depressed the amount set on the keyboard is added in the lower dials.

**3 Minus Bar** The minus bar is used for subtraction. Each time it is depressed the amount set on the keyboard is subtracted from the lower dials.

**4 Repeat and Non-repeat Lever** For addition and subtraction this lever should be in the down position so that the NR symbol is visible; then an amount set on the keyboard is automatically cleared upon each depression of the plus or minus bar. When the lever is in the upper position with the R symbol visible, an amount will be retained on the keyboard until cleared by the operator. In fully automatic operations the lever can be in either the upper or lower position for its functioning is automatically controlled.

**5 Lower Dials** The lower dials register results in addition and subtraction and the product in multiplication. A dividend is registered in the lower dials before starting a division.

**6 Right Upper Dials** The Monroe 8N has a double set of upper counting, or proof, dials. The right-hand set registers the multiplier in multiplication and the quotient in division. They are carry-over dials which give a true count above 9 and serve as an item counter in addition. They also are used for accumulating multipliers or quotients.

**7 Left Upper Dials** The left-hand or Series 3 dials, an exclusive feature of the Monroe, also register multipliers and quotients. These dials, which show multipliers in black figures and quotients in red figures, are not the carry-over type, for they are intended for certain applications in which this manner of functioning is very useful.

**8 Clear Keys** In most operations of the 8N, amounts are cleared from the dials and keyboard automatically. When manual clearance is required it is performed by depressing one or more of the group of three keys. The key marked UPPER clears the upper dials; the one marked LOWER clears the lower dials; the bottom one marked KB CLEAR clears the entire keyboard. They can be depressed separately or jointly. All are electrically operated so only the slightest pressure of the finger is required. When an amount is to be locked on the keyboard, the KB CLEAR is held down while setting the amount on the keyboard. An amount thus locked can be cleared from the keyboard by depressing other keys in the same columns and then depressing the KB CLEAR.

**9 Half-cent Control** The eighth lower dial of the Monroe 8N-213 is equipped for automatic half-cent adjustment of results. After setting a 5 in the keyboard column directly under the eighth lower dial, the operator holds down the half-cent control and depresses the plus bar. Thereafter every time the lower dials are cleared the eighth dial clears to 5 instead of 0. When this automatic function of the 5 is no longer needed and that dial is to be returned to normal operation, the operator sets a 5 in the keyboard column directly under the eighth lower dial, holds down the half-cent control, and depresses the minus bar.

**10 Upper Dials Lock** This lever controls the clearance of the right upper dials. Normally it remains in the upper position; when shifted down the figures in the right upper dials will not clear and amounts can be accumulated. Because of their use a lock is not required for the left upper dials.

**11 Lower Dials Locks** There are two locks that control the clearing of the lower dials. When both of the levers are up, as in most operations, all the lower dials can be cleared. When both levers are down all the lower dials are locked against clearance. When the left of the two levers is shifted down, all lower dials from the 10th to the 21st inclusive are locked and do not clear, but the dials from the 1st to the 9th inclusive can be cleared. When the right-hand lever is shifted down, the right-hand section of the lower dials from 1 to 9 inclusive is locked against clearance and the dials from 10 to 21 inclusive can be cleared.

**12 Carriage Shift Keys** In most operations of the Monroe 8N, the shifting of the carriage is automatic. At other times the carriage can be shifted by depressing these keys which are marked with arrows to designate the direction in which the carriage moves. Depression of the upper key shifts the carriage to the left. Depression of the lower key shifts the carriage to the right.

**13 Tab Stops** When any of the eight tab stops is depressed it determines the position in which the carriage will stop in multiplication and division. A depressed tab stops the carriage when it is shifting in either direction. When two tab stops are required, both should be depressed simultaneously. Tab stops are released by depressing the yellow clear tab at the right.

**14 Enter Multiplier Key** Depressing the ENTER MULTIPLIER key transfers an amount set on the keyboard to be used as a multiplier into the multiplier dials; the amount automatically clears from the keyboard. When the carriage is in any but the first position, depressing the ENTER MULTIPLIER key also returns the carriage to the extreme left or first position. For squaring an amount that has been set on the keyboard, the ENTER MULTIPLIER is depressed and held down until the machine cycle is fully completed; the amount will still remain on the keyboard.

**15 Multiplier Dials** When a multiplier is set in the machine as just described, it is registered in the multiplier dials. An amount in the lower dials to be used as a multiplier can be automatically transferred from the lower dials to the multiplier dials.

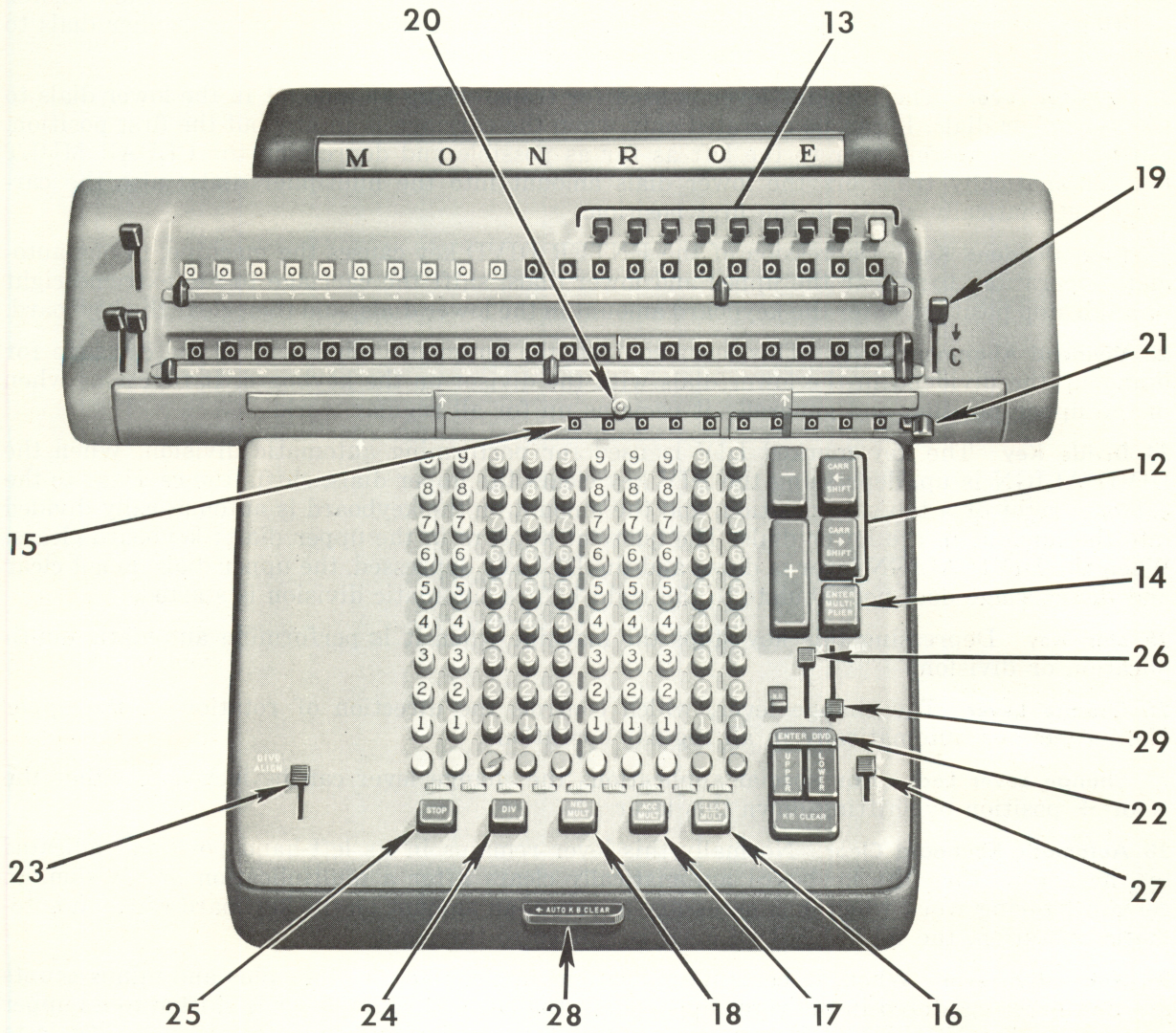
**16 Clear Multiply Key** After an amount has been registered in the multiplier dials when the carriage is in the first position, depressing the CLEAR MULT key clears the upper and lower dials and the amount set on the keyboard is automatically multiplied by the amount in the multiplier dials. At the completion of the multiplication the keyboard clears and the carriage returns to either the first position or to a tab stop position; the multiplier is registered in the upper dials and the result in the lower dials. If the carriage is in any but the first position and the CLEAR MULT is depressed, the upper and lower dials clear and the carriage returns to a tab stop or the first position.

**17 Accumulative Multiply Key** The ACC MULT key is used for accumulative multiplication and operates only with the carriage in the first position. When the key is depressed the amount on the keyboard is multiplied by the amount in the multiplier dials and the product is automatically added into an amount previously registered in the lower dials. The multiplier is also added to whatever amount may be in the right upper dials.

**18 Negative Multiply Key** The NEG MULT key also operates only when the carriage is in the first position. When it is depressed the amount on the keyboard is multiplied negatively by the amount in the multiplier dials and subtracted from whatever amount may be in the lower dials. The multiplier is registered in the upper dials negatively unless the change lever is in the ÷ position.

**19 Constant Multiplier Lever** To retain an amount in the multiplier dials for using it as a constant multiplier, this lever is shifted down. When the constant factor is no longer required the lever is moved to the upper position before the last multiplication of a series is performed. When the constant lever is down other figures cannot be entered in the multiplier dials.

**20 Transfer Slide** The principal function of the transfer slide is to control the automatic shifting and positioning of the carriage. The slide is moved to the right or left by first pulling out the small knob, shifting the slide to the desired position, and then releasing the knob and




12 Carriage Shift Keys
13 Tab Stops
14 Enter Multiplier Key
15 Multiplier Dials
16 Clear Multiply Key
17 Accumulative Multiply Key
18 Negative Multiply Key
19 Constant Multiplier Lever
20 Transfer Slide
21 Transfer Lever
22 Enter Dividend Key
23 Dividend Alignment Lever
24 Divide Key
25 Stop Key
26 Change Lever
27 Change Lever Lock
28 Automatic Keyboard Clear
29 Non-entry Lever

seating it. The green strip at the right-hand end of the slide serves as a guide to the transfer position; for example, when the slide is positioned so that there are four multiplier dials to the right of the green strip the transfer is said to be in the 4 position.

**21 Transfer Lever** The transfer lever is used for transferring an amount in the lower dials to the multiplier dials. It can be operated only when the carriage is in any but the first position. Shifting the transfer lever to the left as far as possible and depressing the CLEAR MULT key automatically transfers the lower dials amount into the multiplier dials; also the carriage automatically shifts to the first position.

**22 Enter Dividend Key** Depression of the ENTER DIVD key causes the machine, in one automatic operation, to: Clear the upper and lower dials, shift the carriage to the extreme right or a tab stop position, enter the keyboard amount in the lower dials, and then clear the keyboard.

**23 Dividend Alignment Lever** This lever controls the automatic alignment of the carriage for lining up the dividend in the lower dials with the divisor on the keyboard. It functions when in the upper position and generally it remains up for all regular work.

**24 Divide Key** The key marked DIV is used for performing automatic division. When the DIVD ALIGN is up, depressing the DIV key clears the upper dials, shifts the carriage to the extreme right or a tab stop position, and the amount on the keyboard is automatically divided into the amount in the lower dials. The quotient appears in the upper dials; keyboard clears. When the DIVD ALIGN lever is down and the DIV key depressed, the upper dials do not clear and the carriage does not tabulate to the right before automatic division is started.

**25 Stop Key** Depressing this key stops the machine when it is performing automatic multiplication or division.

**26 Change Lever** The change lever, which controls the direction of rotation of the upper dials, functions automatically in all regular operations.

**27 Change Lever Lock** For special applications the change lever can be locked in either the × or ÷ position by shifting down this lever.

**28 Automatic Keyboard Clear** Normally this lever remains in its left-hand position, indicated by the arrow, then the keyboard automatically clears after a multiplication or division. In certain figuring work when it is necessary to retain an amount on the keyboard after an automatic operation, the AUTO KB CLEAR is shifted to the right.

**29 Non-entry Lever** When, in certain applications, multipliers or other plus and minus counts are not to be registered in the right upper dials, the NON-ENTRY lever is shifted to its upper position. If the NON-ENTRY lever is in the upper position and the change lever (26) is to be locked in the × position, moving the change lever lock (27) down automatically causes the NON-ENTRY control to shift down into its normal position; when the NON-ENTRY lever is returned to the upper position it automatically releases the change lever lock.

## DECIMALS

One of the greatest advantages of the Monroe Calculator is its automatic decimal system. For the guidance of the operator the dials have movable markers and there are decimal markers between the columns of keys to indicate the position of the decimal point on the keyboard. Generally once these markers are set the Monroe is ready for any and all calculations; changing from one problem to another does not require changing decimals. The system relieves the operator of any concern over the accuracy of decimal points, for with amounts set around a constant decimal answers appear correctly pointed off automatically.

### Monroe Rule

The setting of decimals is the same for all Monroe Calculators and is based on the one, easily remembered, simple rule.

**Keyboard Decimal + Upper Dials Decimal = Lower Dials Decimal**

That is, the number of decimal places in the lower dials is always the total of the decimal places in the upper dials plus the decimal places on the keyboard. The same decimal setting is used in both the right and left upper dials, except in rare cases. If the Monroe rule is followed, results are produced automatically around the correct preset decimal markers.

The operator, by examining the work in hand, quickly determines the maximum number of decimal places involved and the maximum number of decimal places desired in the result. A few pointers are given that will aid in setting the decimals for most kinds of ordinary figuring work.

#### Decimals in Addition and Subtraction

| | |
|---|---|
| Keyboard | Set decimal to handle largest number of decimal places in the numbers to be added. |
| Upper Dials | No decimal required. |
| Lower Dials | Set decimal same as keyboard.<br>Keyboard decimal + Upper dials decimal = Lower dials decimal. |

#### Decimals in Multiplication

| | |
|---|---|
| Keyboard | Examine the multiplier and multiplicand and determine which has the larger number of decimal places. Set the keyboard decimal to accommodate this number of decimal places. |
| Upper Dials | Set decimal same as keyboard. |
| Lower Dials | Keyboard decimal + Upper dials decimal = Lower dials decimal. |

#### Decimals in Division

| | |
|---|---|
| Keyboard | Examine the divisor and dividend and determine which has the larger number of decimal places. Set the keyboard decimal to accommodate this number of decimal places. |
| Upper Dials | Always decide how many decimal places are required in the answer and set the decimal to one more than this (to permit rounding off if required). |
| Lower Dials | Keyboard decimal + Upper dials decimal = Lower dials decimal. |

Since all the arithmetical processes are frequently called for in a single figuring job of engineering, decimals can be set to provide for performing any combination of these and for

changing readily from one to another. A standard decimal set-up for the Monroe 8N that is preferred by many operators is: Keyboard 5, upper dials 5, lower dials 10. In conjunction with this decimal set-up, a tab stop is set at 6 and the transfer slide at 5.

## CONTROL SETTINGS

The operator of the Monroe can keep to a minimum the changing of controls by deciding an arrangement that will take care of a major part of the figuring work being performed. In the instructions that follow and in the applications, the recommended settings of the controls for the desired method of solution are given in the program directions at the beginning of the step-by-step instructions.

### Regular Set-up of Controls

The operating controls remain in a regular or normal position unless otherwise stated in the program. These regular operating positions can be summed up briefly as follows:

| KEY OR CONTROL | REGULAR OPERATING POSITION |
|---|---|
| Repeat and non-repeat lever | Non-repeat |
| Change lever | × position |
| Change lever lock | Up |
| Non-entry lever | Down |
| Automatic keyboard clear | Left position |
| Dividend alignment lever | Up |
| Dials locks | Up |
| Constant multiplier lever | Up |
| Transfer slide | To extreme right |
| Tab stops | None |

## INSTRUCTIONS FOR THE ARITHMETICAL PROCESSES

The directions for performing the fundamental arithmetic processes that follow are given step by step and in detail. After becoming familiar with them, the operator can readily apply the Monroe to the general run of figuring work.

Before starting to use the Monroe the operator should quickly check the machine to be sure that every control is in its normal operating position and the dials clear. The standard preset program for decimals and controls can be used throughout for all the examples, except when otherwise noted.

**Program** Decimals: Keyboard 5 Tab at 6
Upper Dials 5 Transfer Slide 5
Lower Dials 10

### Addition

**Example**

105.032
55.00
14.96
174.992

*Step 1* Depress simultaneously the upper, lower, and keyboard clear keys and ENTER DIVD which clears dials and keyboard and automatically shifts the carriage so the decimals on the keyboard and the lower dials are in alignment.

*Step 2* Set 105.032 on the keyboard at decimal. Depress plus bar.

*Step 3* Set 55. on the keyboard and depress plus bar.

*Step 4* Set 14.96 on the keyboard and depress plus bar.

*Results* Lower dials 174.992, Total
Upper dials 3, Number of items added

### Subtraction

**Example**

2004.76
— 455.0054
1549.7546

*Step 1* With machine clear, set 2004.76 on the keyboard at decimal. Depress plus bar.

*Step 2* Set 455.0054 on the keyboard and depress minus bar.

*Result* Lower dials 1549.7546, Remainder

### Multiplication

**Example** $611.805 \times 3.21 = 1963.89405$

*Step 1* Set 3.21 on the keyboard (the smaller factor is generally used as the multiplier and set first). Depress ENTER MULTIPLIER.

*Step 2* Set 611.805 on the keyboard. Depress CLEAR MULT.

*Result* Lower dials 1963.89405, Product
Upper dials 3.21 Multiplier

### Accumulative Multiplication

**Example** 125.33 × 2.42 = 303.2986
67.803 × 40.11 = 2719.57833
42.53 3022.87693

*Step 1* Set 2.42 on the keyboard and depress ENTER MULTIPLIER.

*Step 2* Set 125.33 on the keyboard and depress CLEAR MULT key. *Result* Lower dials 303.2986.

*Step 3* Set 40.11 on keyboard and depress ENTER MULTIPLIER.

*Step 4* Set 67.803 on keyboard and depress ACC MULT.

*Results* Lower dials 3022.87693, Accumulated products
Upper dials 42.53 Accumulated multipliers

### Negative Multiplication

**Example** (20.1 × 145.082) − (3.00 × 48.57) = 2770.4382

*Step 1* Set 20.1 on the keyboard and depress ENTER MULTIPLIER.

*Step 2* Set 145.082 on the keyboard and depress CLEAR MULT. *Result* Lower dials 2916.1482.

*Step 3* Set 3.00 on keyboard and depress ENTER MULTIPLIER.

*Step 4* Set 48.57 on keyboard and depress NEG MULT.

*Result* Lower dials 2770.4382
Upper dials 17.1 (20.1 − 3.00)

### Multiplication with Constant

**Example** 34.5 × 2.25 = 77.625
18.0 × 2.25 = 40.500
59.4 × 2.25 = 133.650

*Step 1* Set 2.25 on the keyboard and depress ENTER MULTIPLIER. Move down constant multiplier lever.

*Step 2* Set 34.5 on the keyboard and depress CLEAR MULT.

*Result* Lower dials 77.625

*Step 3* Set 18. on the keyboard and depress CLEAR MULT.

*Result* Lower dials 40.500

*Step 4* Raise constant multiplier lever. Set 59.4 on the keyboard and depress CLEAR MULT.

*Result* Lower dials 133.650

### Multi-factor Multiplication

**Example** 14 × 126.2 × 58.04 = 102545.072

*Step 1* Set 14 on the keyboard and depress ENTER MULTIPLIER.

*Step 2* Set 126.2 on the keyboard and depress CLEAR MULT. *Result* Lower dials 1766.8.

*Step 3* Shift transfer lever to left as far as it will go. Depress CLEAR MULT key, transferring result of first multiplication to multiplier dials.

*Step 4* Set 58.04 on keyboard. Depress CLEAR MULT.

*Result* Lower dials 102545.072

Steps 3 and 4 can be repeated for any number of multiplications.

### Squaring

**Example** $(13.52)^2 = 182.7904$

*Step 1* Set 13.52 on the keyboard at decimal. Depress and hold down ENTER MULTIPLIER key until cycle is completed, which retains amount on the keyboard.

When squaring with the Model 8N-1, the squaring lever is shifted to its lower position so that the factor on the keyboard is automatically retained, thus making it unnecessary to hold down the ENTER MULTIPLIER key as described in Step 1 above.

*Step 2* Depress CLEAR MULT key.

*Result* Lower dials 182.7904 $(13.52)^2$

### Squaring and Accumulating

**Example** $(18)^2 + (16.23)^2 = 587.4129$

*Step 1* Set 18 on the keyboard. Depress ENTER MULTIPLIER key and hold down.

*Step 2* Depress CLEAR MULT.

*Step 3* Set 16.23 on the keyboard. Depress ENTER MULTIPLIER and hold down.

*Step 4* Depress ACC MULT.

*Result* Lower dials 587.4129

The operations of Steps 3 and 4 are continued for securing the sum of any numbers of squares.

### Squaring and Deducting

**Example** $(25.1)^2 - (14.5)^2 = 419.76$

*Step 1* Set 25.1 on the keyboard. Depress ENTER MULTIPLIER key and hold.

*Step 2* Depress CLEAR MULT.

*Step 3* Set 14.5 on the keyboard. Depress ENTER MULTIPLIER and hold.

*Step 4* Depress NEG MULT.

*Result* Lower dials 419.76

### Cube and Higher Powers

**Example** $(19.7)^3 = 7645.373$

*Step 1* Move AUTO KB CLEAR to the right. Set 19.7 on the keyboard. Depress and hold down ENTER MULTIPLIER.

*Step 2* Depress CLEAR MULT to square.

*Step 3* Move transfer lever to the left as far as it will go. Depress CLEAR MULT twice.

*Result* Lower dials 7645.373

An amount can be raised to any higher power by continuing this method of transfer multiplication.

### Division

**Example** $219.65 \div 14.325 = 15.33333$

*Step 1* Set dividend, 219.65, on the keyboard at decimal. Depress ENTER DIVD key.

*Step 2* Set 14.325 on the keyboard. Depress DIV key.

*Result* Upper dials 15.33333

## Percentage of Increase or Decrease

Percentages are determined by division and due to its exclusive Series 3 upper dials the Monroe shows automatically whether a percentage is an increase or a decrease. The operator can tell immediately by remembering that answers in black figures in the dials indicate an increase and red figures a decrease, according to the following simple rule.

> If a black 1 precedes the figures in the left upper dials the percentage is read as an ***increase*** in the right upper dials.
>
> If all figures are red in the left upper dials the percentage is read as a ***decrease*** in those dials.

**Example**

| THIS YEAR | LAST YEAR | % DECREASE | % INCREASE |
|---|---|---|---|
| 958.22 | 723.90 | | 32.37 |
| 850.00 | 1047.16 | 18.83 | |
| 1127.65 | 959.35 | | 17.54 |

**Program**

| | | | |
|---|---|---|---|
| Decimals: | Keyboard | 5 | Tab at 6 |
| | Upper Dials | 5 | Transfer Slide 5 |
| | Lower Dials | 10 | Repeat Control On |
| | | | Change Lever Locked |
| | | | Divd Align Down |

*Step 1* Set 1.00 on the keyboard at decimal. Depress ENTER MULTIPLIER key. Move constant lever down.

*Step 2* Set this year's figure, 958.22, on the keyboard. Depress NEG MULT.

*Step 3* Set last year's figure, 723.90, on the keyboard. Depress and hold down plus bar until machine stops. Depress DIV key. Because a black 1 precedes figures in the left upper dials, answer is read in right upper dials.

*Result* Right upper dials .32370 or 32.37% Increase

*Step 4* Clear dials. Set 850.00 on keyboard. Depress NEG MULT.

*Step 5* Set 1047.16 on keyboard. Depress and hold down plus bar until machine stops. Depress DIV key. Because all figures are in red in the left upper dials, answer is read in those dials.

*Result* Left upper dials .18828 or 18.83% Decrease

Clear machine and repeat Steps 4 and 5 for next percentage.

*Result* Right upper dials .17544 or 17.54% Increase

When using this method to find the percentage of increase or decrease in a series of amounts, the constant lever is moved up before starting Step 4 for the final calculation of the series. Thus the 1 will be automatically removed from the multiplier dials.

## Finding a Reciprocal

The reciprocal of a number is 1 divided by that number. For example, the reciprocal of 4 is ¼ or .25.

**Example** The reciprocal of 432 is .002314815.

**Program**

| | | | |
|---|---|---|---|
| Decimals: | Keyboard | 0 | No Tab Stops |
| | Upper Dials | 0 | |
| | Lower Dials | 0 | |

*Step 1* Enter the dividend by setting 1 on the extreme left of the keyboard (in tenth column). Depress ENTER DIVD.

*Step 2* Set 432 on extreme left of the keyboard. Depress DIV key.

*Result* Upper dials 23148148 or .002314815

Decimals are pointed off in reciprocals according to the following rule:

The reciprocal of any whole number or of a whole number and decimal is always a decimal. Prefix as many zeros to the reciprocal as there are whole numbers in the divisor, ***less one.***

The reciprocal of any decimal is always a whole number or a whole number and a decimal. Point off as many whole numbers in the reciprocal as there are ciphers in the divisor, ***plus one.***

## Percentage Distribution by Reciprocal

**Example**

| GROUP | VALUE | PER CENT |
|---|---|---|
| A | 1164 | 13.39 |
| B | 1629 | 18.74 |
| C | 2371 | 27.28 |
| D | 3528 | 40.59 |
| | 8692 | |

**Program** Decimals: Keyboard 2
Upper Dials 10
Lower Dials 12

*Step 1* Find the reciprocal of 8692 which is .0001150483.

*Step 2* Set .0001150483 on the keyboard and depress ENTER MULTIPLIER. Move down constant lock lever as reciprocal is to be used as constant multiplier.

*Step 3* Set value for Group A, 1164.00, on the keyboard. Depress CLEAR MULT.

*Result* Lower dials .1339162212 or 13.39% Group A is of total

*Step 4* Set value for B, 1629.00, on the keyboard. Depress CLEAR MULT.

*Result* Lower dials .1874136807 or 18.74% Group B is of total

Continue Step 4 for determining percentages of Groups C and D. Before making the final multiplication to find Group D, move constant lock lever to upper position.

*Results* Lower dials 27.28% Group C
Lower dials 40.59% Group D

## Square Root

The Monroe Simplified Method for Extracting Square Root is rapid and gives results with accuracy to five significant figures with an error of less than 5 in the sixth figure. It makes use of a table of factors which is reproduced in the appendix where instructions are also given.

# MONROE MACHINE METHODS

# CIVIL ENGINEERING CALCULATIONS

## PART II

## APPLICATIONS

## Degrees, Minutes, and Seconds

A few examples will readily show how simply and easily degrees, minutes, and seconds can be added or subtracted on the Monroe Calculator.

**Program** Decimals: Keyboard 7-3
Upper Dials 0
Lower Dials 7-3

The decimals are used as a guide to differentiate degrees, minutes, and seconds. The degrees are set to the left of the seventh decimal, minutes at the third decimal, and seconds on the extreme right of the keyboard.

### ADDITION

**Example 1**

17° 10′ 2″
18′ 31″
17° 28′ 33″

*Step* 1 Set 17° 10′ 2″ on the keyboard as 17.0010.002 and depress plus bar.

*Step 2* Set 18′ 31″ on the keyboard as 18.031 and depress plus bar.

*Result* Lower dials 17.0028.033 or 17° 28′ 33″

**Example 2**

73° 8′ 45″
89° 12′ 59″
162° 21′ 44″

*Step 1* Set 73° 8′ 45″ on the keyboard. Depress plus bar.

*Step 2* Set 89° 12′ 59″ on the keyboard. Depress plus bar. *Result* Lower dials 162° 20′ 104″.

*Step 3* Set the complement of 60 seconds, .940, on the right of keyboard and depress plus bar.

*Result* Lower dials 162.0021.044 or 162° 21′ 44″

**Example 3**

100° 55′ 21″
19° 59′ 44″
85° 43′ 9″
206° 38′ 14″

*Step 1* Add 100° 55′ 21″, 19° 59′ 44″, and 85° 43′ 9″. *Result* Lower dials 204° 157′ 74″.

*Step 2* Move repeat lever up. Set .940 on right of keyboard. Depress plus bar once to add.

*Step 3* Change keyboard set-up to .9940.000. With plus bar make continued additions until the minutes in the lower dials are less than 60.

*Result* Lower dials 206.0038.014 or 206° 38′ 14″

These examples illustrate the principle of using the complement and show that the plus bar must be depressed as many times as may be required to reduce the groups of figures in the lower dials representing seconds and minutes to less than 60.

## SUBTRACTION

**Example 1**

310° 37′ 3″
−195° 21′ 29″
115° 15′ 34″

*Step 1* Set 310° 37′ 3″ on the keyboard and depress plus bar to register in lower dials.

*Step 2* Set 195° 21′ 29″ on the keyboard and depress minus bar to subtract. *Result* Lower dials 115° 15′ 974″

*Step 3* Set .940 on right of keyboard and subtract.

*Result* Lower dials 115.0015.034 or 115° 15′ 34″

**Example 2**

360°
− 54° 39′ 20″
− 81° 9′ 50″
−113° 0′ 7″
111° 10′ 43″

*Step 1* Set 360. at seventh decimal and depress plus bar to register in lower dials.

*Step 2* Set 54° 39′ 20″ on the keyboard and subtract. Subtract 81° 9′ 50″ and 113° 0′ 7″.

*Result* Lower dials 111° 9951′ 923″

*Step 3* Move repeat lever up. Set .940 on right of keyboard and subtract until the seconds in lower dials are less than 60.

*Step 4* Change keyboard set-up to .9940.000 and subtract until the minutes are less than 60.

*Result* Lower dials 111.0010.043 or 111° 10′ 43″

# Interpolations

## NATURAL FUNCTIONS

These Monroe methods for making interpolations are extremely useful to the engineer because they make it possible to interpolate in either direction without copying intermediate results.

Since interpolation is most always incidental to other figuring work, some engineers do not change the decimal set-up of the Monroe Calculator and disregarding decimal points they treat all figures as whole numbers and only point off in the final interpolated results when writing them down.

### To Find the Function

**Formula** $f(x) = \dfrac{Sa - Sb + b\,(60)}{60}$

**Example 1** Find the sin 11° 10′ 51″

Sin 11° 11′ = .1939490, a
Sin 11° 10′ 51″ = x
Sin 11° 10′ = .1936636, b
S = 51″

**Program** Decimals: Keyboard 8 AUTO KB CLEAR to Right
Upper Dials 8
Lower Dials 16

*Step 1* Multiply .1939490, a, by 51, S. Multiply negatively 51, S, by .1936636, b.

*Step 2* Multiply accumulatively .1936636, b, by 60. Divide by 60.

*Result* Upper dials .1939062, Sin 11° 10′ 51″

**Example 2** Find the cos 11° 10′ 51″

Cos 11° 11′ = .9810116, a
Cos 11° 10′ 51″ = x
Cos 11° 10′ = .9810680, b

*Step 1* Multiply .9810116, a, by 51, S. Multiply negatively 51, S, by .9810680, b.

*Step 2* Multiply accumulatively .9810680, b, by 60. Divide by 60.

*Result* Upper dials .9810201, Cos 11° 10′ 51″

### To Find the Angle

**Formula** $f(x) = \dfrac{60\,(x - a)}{(b - a)}$

**Example 1** Find the angle when sin is .5555555

Sin 33° 45′ = .5555702, b
Sin x = .5555555
Sin 33° 44′ = .5553283, a

**Program** Decimals: Keyboard 8 Tab at 9
Upper Dials 8 Transfer Slide 8
Lower Dials 16 Repeat Control On

*Step 1* Set .5555702, b, on keyboard. Depress ENTER DIVD to register in lower dials. Sub-

tract .5553283, a. *Result* Lower dials .0002419, b — a, which is noted.

*Step 2* Clear dials. Subtract .5553283, a, and add .5555555, x.

*Step 3* Clear keyboard. Transfer and multiply by 60. Divide by .0002419, b — a.

*Result* Upper dials 56.35 or 56″. Therefore the angle is 33° 44′ 56″

**Example 2** Find the angle of which the cos is .8300000

Cos 33° 54′ = .8300123, b
Cos x = .8300000
Cos 33° 55′ = .8298500, a

*Step 1* Set .8300123, b, on the keyboard. Depress ENTER DIVD to register in lower dials. Subtract .8298500, a. *Result* Lower dials .0001623, b — a, which is noted.

*Step 2* Do not clear dials. Add .8298500, a, and subtract .8300000, x.

*Step 3* Clear keyboard. Transfer and multiply by 60. Divide by .0001623, b — a.

*Result* Upper dials 4.55 or 5″. Therefore the angle is 33° 54′ 05″

## LOGARITHMS

### To Find the Log

**Formula** Log (x) = Db — Da + a

In these examples six-place tables are used. The characteristics are disregarded.

**Example 1** Find the log of 333333

Log 3334 = .522966, b
Log 333333 = x
Log 3333 = .522835, a
D = .33

**Program**

| | | | |
|---|---|---|---|
| Decimals: | Keyboard | 8 | Tab at 9 |
| | Upper Dials | 8 | Transfer Slide 8 |
| | Lower Dials | 16 | AUTO KB CLEAR to Right |

*Step 1* Multiply .522966, b, by .33, D.

*Step 2* Multiply negatively .33, D, by .522835, a. Add .522835, a.

*Result* Lower dials .522878, Log 333333 disregarding the characteristic

**Example 2** Find the log of 67582986

Log 6759 = .829882, b
Log 67582986 = x
Log 6758 = .829818, a
D = .2986

*Step 1* Multiply .829882, b, by .2986, D.

*Step 2* Multiply negatively .2986, D, by .829818, a. Add .829818, a.

*Result* Lower dials .829837, Log 67582986 disregarding the characteristic

### To Find the Number

**Formula** $\text{Log } (x) = \dfrac{(x - a)}{(b - a)}$

**Example** Find the number represented by log .555555

Log 3594 = .555578, b
Log x = .555555
Log 3593 = .555457, a

**Program** Decimals: Keyboard 8 Tab at 9
Upper Dials 8 Repeat Control On
Lower Dials 16

*Step 1* Set .555578, b, on keyboard. Depress ENTER DIVD to register in lower dials. Subtract .555457, a. *Result* Lower dials .000121, b − a, which is noted.

*Step 2* Clear dials. Subtract .555457, a, and add .555555, x.

*Step 3* Clear keyboard and divide by .000121, b − a.

*Result* Upper dials .8099

The result, .8099 combined with 3593 gives the number 35938099. The decimal point is determined by the characteristic.

# Bearing and Distance of Course

Here are fast, simple, and convenient methods for calculating the course and distance between two stations having plane coordinates by means of the Monroe Calculator used in conjunction with trigonometric tables*.

**Example 1**

| STATION | COORDINATES LATITUDE | DEPARTURE |
|---|---|---|
| 5062 | 10,027.37 N | 12,980.63 E |
| 5247 | 9,648.83 N | 14,469.63 E |
| Difference | 378.54 | 1,489.00 |

Departure difference is greater than the latitude difference; hence 5247 is south and east of 5062 (Figure 1).

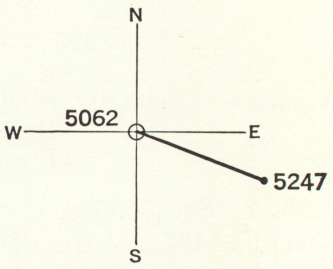



FIGURE 1

**Program** Decimals: Keyboard 5 Tab at 9
Upper Dials 5- 8
Lower Dials 10-13

## BEARING

$$\text{Cot of bearing} = \frac{\text{Latitude difference}}{\text{Departure difference}} = \frac{378.54}{1489.00} = .25422431$$

*Step 1* Set 378.54 on the keyboard at decimal and depress ENTER DIVD.

*Step 2* Set 1489.00 on the keyboard and depress DIV. *Result* Upper dials .25422431, Cot of bearing.

*Result* From table S 75° 44′ 10.27″ E, Bearing

## DISTANCE

$$\text{Distance} = \sqrt{(\text{Latitude difference})^2 + (\text{Departure difference})^2} = \sqrt{(378.54)^2 + (1489.00)^2} = 1{,}536.364$$

*Step 1* Set 378.54 on the keyboard at decimal. Depress and hold down ENTER MULTIPLIER until cycle is completed, which retains the amount on the keyboard.

*Step 2* Depress CLEAR MULT.

*Step 3* Set 1489.00 on the keyboard. Depress ENTER MULTIPLIER and hold down.

*Step 4* Depress ACC MULT. *Result* Lower dials 2360413.5316, Sum of squares.

*Step 5* Find the square root of 2360413.5316 by the Monroe Simplified Method†.

*Result* 1,536.364, Distance

$$\text{Check: Distance} = \frac{\text{Departure difference}}{\text{Sin bearing}} = \frac{1489}{.96917153} = 1536.364$$

Station 5062 to Station 5247: South 75° 44′ 10.27″ East 1536.364 Feet

**Example 2**

| STATION | COORDINATES LATITUDE | DEPARTURE |
|---|---|---|
| 5101 | 8,307.23 N | 8,392.13 E |
| 5235 | 11,064.14 N | 10,097.97 E |
| Difference | 2,756.91 | 1,705.84 |

Total latitude difference is greater than the total departure difference; hence 5235 is north and east of 5101 (Figure 2).

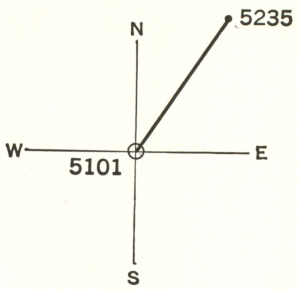


FIGURE 2

**Program** Same

## BEARING

$$\text{Tan of bearing} = \frac{\text{Departure difference}}{\text{Latitude difference}} = \frac{1705.84}{2756.91} = .61875070$$

*Step 1* Set 1705.84 on the keyboard at decimal. Depress ENTER DIVD.

*Step 2* Set 2756.91 on the keyboard. Depress DIV. *Result* Upper dials .61875070, Tan of bearing.

*Result* From table N 31° 44′ 49.84″ E, Bearing

## DISTANCE

$$\text{Distance} = \sqrt{(2756.91)^2 + (1705.84)^2} = 3241.981$$

*Step 1* Set 2756.91 on the keyboard at decimal. Depress and hold down ENTER MULTIPLIER until cycle is completed.

*Step 2* Depress CLEAR MULT.

*Step 3* Set 1705.84 on the keyboard. Depress ENTER MULTIPLIER and hold down.

*Step 4* Depress ACC MULT. *Result* Lower dials 10510442.8537.

*Step 5* Find the square root of 10510442.8537 by Monroe Simplified Method.

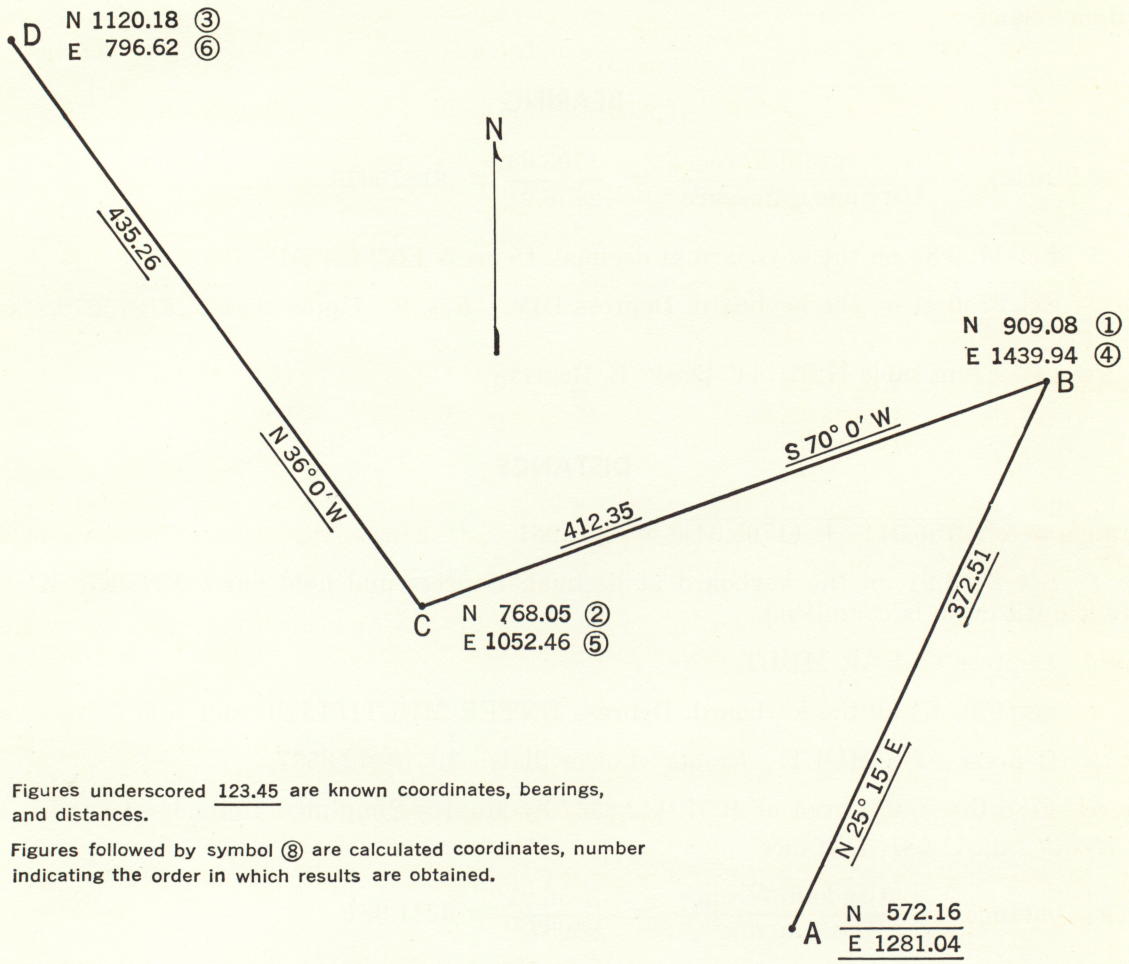
*Result* 3,241.981, Distance

$$\text{Check: Distance} = \frac{\text{Latitude difference}}{\text{Cos bearing}} = \frac{2756.91}{.8503781} = 3241.981$$

Station 5101 to Station 5235: North 31° 44′ 49.84″ East 3241.981 Feet

*"Natural Trigonometric Functions," published by the Monroe Company, is a handy book designed especially for the engineer who uses the Monroe Calculator. The eight-place tables are arranged for rapid solution without using logarithms. The values for all functions of each degree are given on a single page; the second differences to ten decimal places are printed adjacent to the minute values and are read directly without interpolating.

†For instructions see Monroe Form 1191-S or Form 1228-S.




Figures underscored 123.45 are known coordinates, bearings, and distances.

Figures followed by symbol ⑧ are calculated coordinates, number indicating the order in which results are obtained.

**FIGURE 3**

# Open Traverse

## COORDINATES

The engineer can easily and accurately find the coordinates of unknown points by means of tables of natural trigonometric functions and a Monroe Calculator.

**Example** In Figure 3, given the coordinates of point A, the bearings and distances of A-B, B-C, and C-D, find the coordinates of points B, C, and D.

| COURSE | BEARING | FUNCTIONS | | DISTANCE | COORDINATES | | POINT |
|---|---|---|---|---|---|---|---|
| | | COS | SIN | | N:S | E:W | |
| A-B | N 25° 15′ E | .90446 | .42657 | 372.51 | N 572.16 | E 1281.04 | A |
| B-C | S 70° 0′ W | .34202 | .93969 | 412.35 | N 909.08 ① | E 1439.94 ④ | B |
| C-D | N 36° 0′ W | .80902 | .58779 | 435.26 | N 768.05 ② | E 1052.46 ⑤ | C |
| | | | | | N 1120.18 ③ | E 796.62 ⑥ | D |
| | | | | | | | |

**Program** Decimals: Keyboard 5 Tab at 6
Upper Dials 5 Transfer Slide 5
Lower Dials 10

All northings and eastings are multiplied positively and accumulated, and all southings and westings are multiplied negatively and deducted. In this manner the traverse can be continued indefinitely for any number of courses.

The circled numbers (e.g. ②) after the results in the following steps tie in with the values in Figure 3 and the table; these numbers also show the sequence in which the desired results are secured.

*Step 1* Set 572.16, north coordinate of A, on the keyboard at decimal and depress ENTER DIVD. Set 372.51, distance A-B, on the keyboard. Multiply accumulatively by .90446, cos A.

*Result* Lower dials 909.08, North coordinate of B ①

*Step 2* Multiply negatively 412.35, distance B-C, by .34202, cos B.

*Result* Lower dials 768.05, North coordinate of C ②

*Step 3* Multiply accumulatively 435.26, distance C-D, by .80902, cos C.

*Result* Lower dials 1120.18, North coordinate of D ③

*Step 4* Set 1281.04, east coordinate of A, on the keyboard and depress ENTER DIVD. Set 372.51, distance A-B, on the keyboard and multiply accumulatively by .42657, sin A.

*Result* Lower dials 1439.94, East coordinate of B ④

*Step 5* Multiply negatively 412.35, distance B-C, by .93969, sin B.

*Result* Lower dials 1052.46, East coordinate of C ⑤

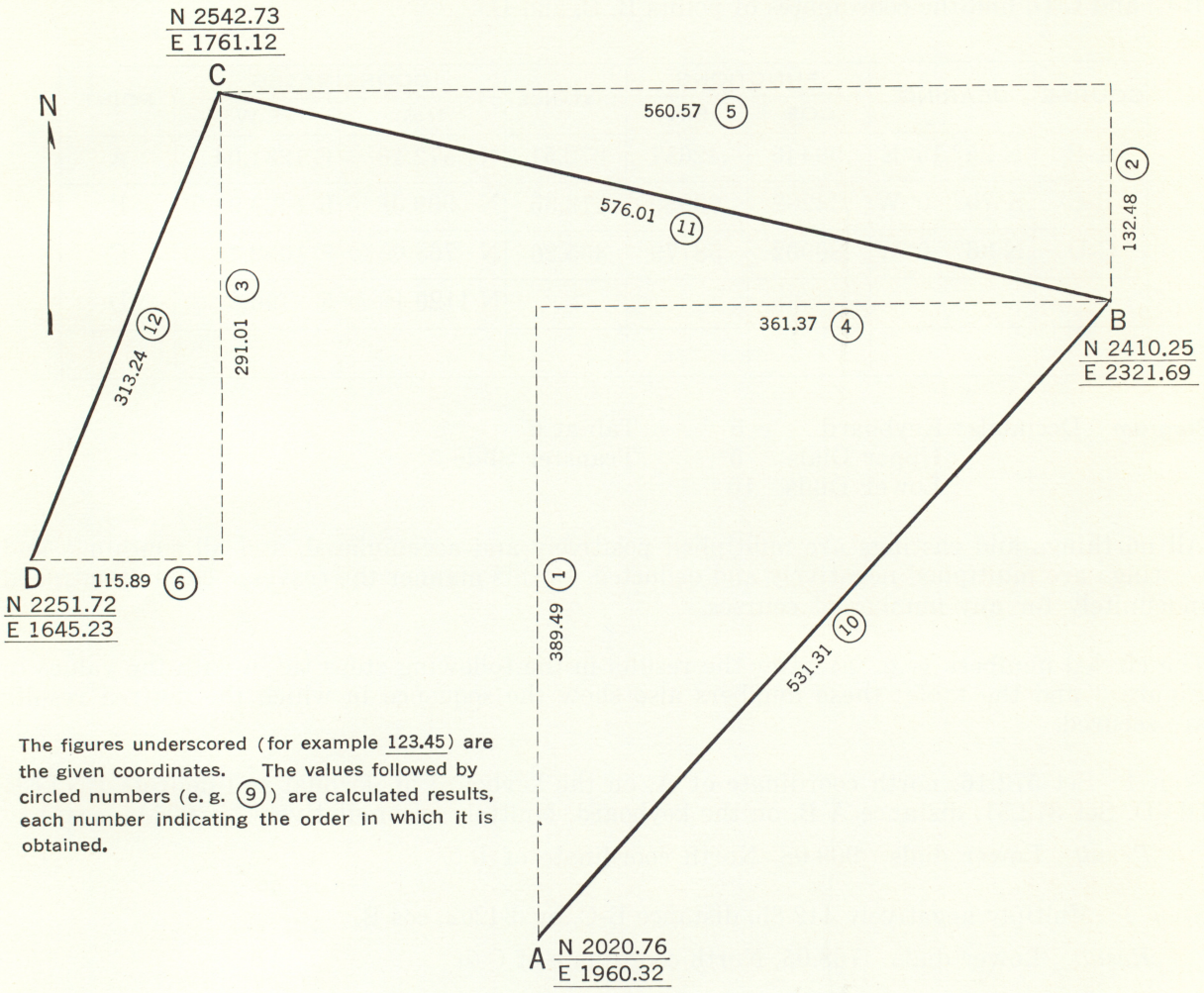
*Step 6* Multiply negatively 435.26, distance C-D, by .58779, sin C.

*Result* Lower dials 796.62, East coordinate of D ⑥

## Open Traverse — Continued

### BEARING AND DISTANCE

**Example** In Figure 4, given the coordinates of points A, B, C, and D, find the bearing of points A, B, and C, and the distances A-B, B-C, and C-D.




The figures underscored (for example 123.45) are the given coordinates. The values followed by circled numbers (e. g. (9)) are calculated results, each number indicating the order in which it is obtained.

**FIGURE 4**

| POINT | COORDINATES | | COORDINATE DIFFERENCES | | FUNCTIONS | | | BEARING |
|---|---|---|---|---|---|---|---|---|
| | N | E | N:S | E:W | TAN | COS | SIN | |
| A | 2020.76 | 1960.32 | | | .92780 ⑦ | .73307 | .68015 | N 42° 51′ 19″ E |
| B | 2410.25 | 2321.69 | N 389.49 ① | E 361.37 ④ | 4.23135 ⑧ | .23000 | .97319 | N 76° 42′ 12″ W |
| C | 2542.73 | 1761.12 | N 132.48 ② | W 560.57 ⑤ | .39823 ⑨ | .92904 | .36997 | S 21° 42′ 50″ W |
| D | 2251.72 | 1645.23 | S 291.01 ③ | W 115.89 ⑥ | | | | |
| | | | | | | | | |

## Coordinate Differences

**Program** Decimals: Keyboard 5 Tab at 6
Upper Dials 5
Lower Dials 10

The coordinate differences are secured with the carriage in the sixth position.

### N:S Coordinate Differences

*Step 1* Subtract 2020.76 from 2410.25.
*Result* Lower dials 389.49, N ①

*Step 2* Subtract 2410.25 from 2542.73.
*Result* Lower dials 132.48, N ②

*Step 3* Subtract 2251.72 from 2542.73.
*Result* Lower dials 291.01, S ③

### E:W Coordinate Differences

*Step 4* Subtract 1960.32 from 2321.69.
*Result* Lower dials 361.37, E ④

*Step 5* Subtract 1761.12 from 2321.69.
*Result* Lower dials 560.57, W ⑤

*Step 6* Subtract 1645.23 from 1761.12.
*Result* Lower dials 115.89, W ⑥

Thus, point B is 389.49 feet north and 361.37 feet east of point A, and so forth.

## Functions and Bearings

*Step 1* Set 361.37, E:W coordinate difference, on the keyboard and depress ENTER DIVD. Set 389.49, N:S coordinate difference, on the keyboard and divide.
*Result* Upper dials .92780 ⑦, Tan of N 42° 51′ 19″ E; Cos .73307; Sin .68015

*Step 2* Divide 560.57, E:W coordinate difference, by 132.48, N:S coordinate difference.
*Result* Upper dials 4.23135 ⑧, Tan of N 76° 42′ 12″ W; Cos .23000; Sin .97319

*Step 3* Divide 115.89, E:W coordinate difference, by 291.01, N:S coordinate difference.
*Result* Upper dials .39823 ⑨, Tan of S 21° 42′ 50″ W; Cos .92904; Sin .36997

## Distances

*Step 1* Set 361.37, E:W coordinate difference, on the keyboard and depress ENTER DIVD. Set .68015, sin 42° 51′ 19″, on the keyboard and divide.
*Result* Upper dials 531.31 ⑩, Distance A-B

*Step 2* Divide 560.57, E:W coordinate difference, by .97319, sin 76° 42′ 12″.
*Result* Upper dials 576.01 ⑪, Distance B-C

*Step 3* Divide 115.89; E:W coordinate difference, by .36997, sin 21° 42′ 50″.

*Result* Upper dials 313.24⑫, Distance C-D

### Checking Calculations

*Step 1* Set 2020.76, north coordinate of A, on the keyboard and depress ENTER DIVD. Multiply accumulatively .73307, cos A, by 531.31, distance A-B.

*Result* Lower dials 2410.25, North coordinate of B

*Step 2* Do not clear machine. Multiply accumulatively .23000, cos B, by 576.01, distance B-C.

*Result* Lower dials 2542.73, North coordinate of C

*Step 3* Do not clear. Multiply negatively .92904, cos C, by 313.24, distance C-D.

*Result* Lower dials 2251.72, North coordinate of D

### Closed Traverse and Area

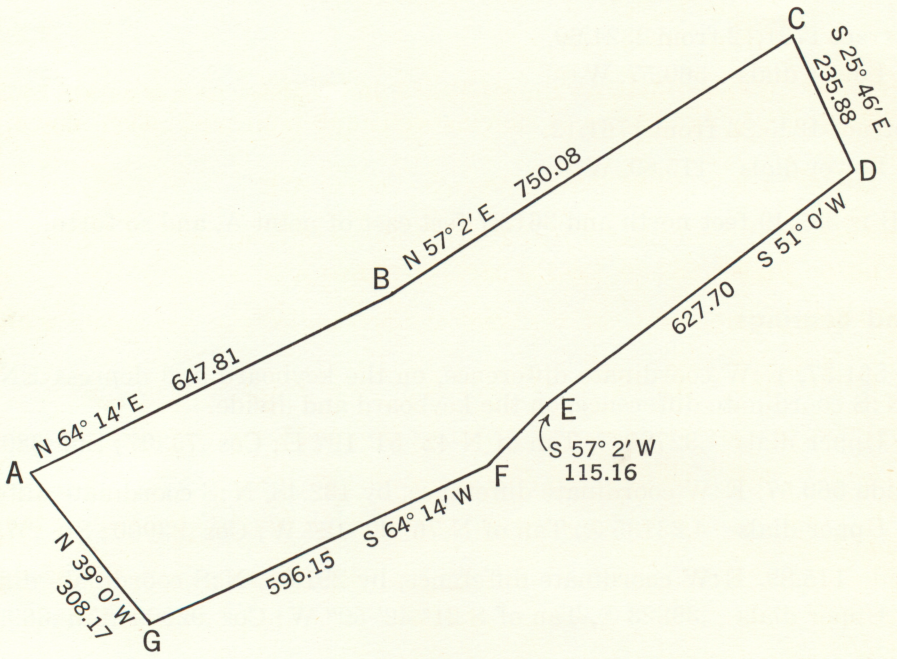



**FIGURE 5**

# Closed Traverse and Area

**Example 1** Given the data for the closed traverse of Figure 5, calculate the latitudes and departures, double meridian distances, and area.

| COURSE | BEARING | DISTANCE | FUNCTIONS | | LATITUDE | | DEPARTURE | | DMD |
|---|---|---|---|---|---|---|---|---|---|
| | | | COS | SIN | N | S | E | W | |
| A-B | N 64° 14′ E | 647.81 | .43471 | .90057 | 281.61 | | 583.40 | | +583.40 |
| B-C | N 57° 2′ E | 750.08 | .54415 | .83899 | 408.16 | | 629.31 | | +1796.11 |
| C-D | S 25° 46′ E | 235.88 | .90057 | .43471 | | 212.43 | 102.54 | | +2527.96 |
| D-E | S 51° 0′ W | 627.70 | .62932 | .77715 | | 395.02 | | 487.82 | +2142.68 |
| E-F | S 57° 2′ W | 115.16 | .54415 | .83899 | | 62.66 | | 96.62 | +1558.24 |
| F-G | S 64° 14′ W | 596.15 | .43471 | .90057 | | 259.15 | | 536.87 | +924.75 |
| G-A | N 39° 0′ W | 308.17 | .77715 | .62932 | 239.49 | | | 193.94 | + 193.94 |
| | | | | | 929.26 | 929.26 | 1315.25 | 1315.25 | |

## LATITUDES AND DEPARTURES

**Program** Decimals: Keyboard 5
Upper Dials 5
Lower Dials 10

*Step 1* Set 647.81, distance A-B, on the keyboard and register as a constant multiplier by depressing ENTER MULTIPLIER and moving constant multiplier lever down. Multiply by .43471, cos A.

*Result* Lower dials 281.61, North latitude, course A-B

*Step 2* Raise constant multiplier lever. Multiply by .90057, sin A.

*Result* Lower dials 583.40, East departure, course A-B

*Step 3* Set 750.08, distance B-C, on the keyboard and register as a constant multiplier. Multiply by .54415, cos B.

*Result* Lower dials 408.16, North latitude, course B-C

*Step 4* Raise constant multiplier lever. Multiply by .83899, sin B.

*Result* Lower dials 629.31, East departure, course B-C

Continue in the same way, multiplying the distances by the functions, securing the latitudes and departures for all the remaining courses.

## AREA

### Double Meridian Distances

**Program** Decimals: Keyboard 5 Tab at 6
Upper Dials 5 Transfer Slide 5
Lower Dials 10 Repeat Control On

The first DMD is equal to the easting of the first course, 583.40.

*Step 1* Depress ENTER DIVD to shift carriage to sixth position so amounts are registered correctly pointed off in the lower dials. Set 583.40, east departure A-B, on the keyboard and add twice.

*Step 2* To the 1166.80 in the lower dials add 629.31, east departure B-C.

*Result* Lower dials 1796.11, Second DMD

*Step 3* Do not clear machine. To the 1796.11 in the lower dials add 629.31, east departure B-C. Add 102.54, east departure C-D.

*Result* Lower dials 2527.96, Third DMD

*Step 4* To the 2527.96 in the lower dials add 102.54, east departure C-D. Subtract 487.82, west departure D-E.

*Result* Lower dials 2142.68, Fourth DMD

*Step 5* From the 2142.68 in the lower dials subtract 487.82, west departure D-E. Subtract 96.62, west departure E-F.

*Result* Lower dials 1558.24, Fifth DMD

*Step 6* From 1558.24 subtract 96.62, west departure E-F. Subtract 536.87, west departure F-G.

*Result* Lower dials 924.75, Sixth DMD

*Step 7* From 924.75 subtract 536.87, west departure F-G. Subtract 193.94, west departure G-A.

*Result* Lower dials 193.94, Seventh DMD

## Calculating Area

All calculations with northings and eastings are positive operations; all calculations with southings and westings are negative operations.

**Program** Decimals: Keyboard 3 Tab at 4
Upper Dials 3 Transfer Slide 3
Lower Dials 6

*Step 1* Multiply 281.61, north latitude A-B, by 583.40, first DMD.

*Step 2* Multiply accumulatively 408.16, north latitude B-C, by 1796.11, second DMD.

*Step 3* Multiply negatively 212.43, south latitude C-D, by 2527.96, third DMD.

*Step 4* Multiply negatively 395.02, south latitude D-E, by 2142.68, fourth DMD.

*Step 5* Multiply negatively 62.66, south latitude E-F, by 1558.24, fifth DMD.

*Step 6* Multiply negatively 259.15, south latitude F-G, by 924.75, sixth DMD.

*Step 7* Multiply accumulatively 239.49, north latitude G-A, by 193.94 seventh DMD. *Result* Lower dials 999223133.9449.

As the result is a negative amount it must be converted to a positive one, as explained in Step 8. When the result of Step 7 is a positive amount, Step 8 is omitted.

*Step 8* Transfer and multiply negatively by 1. *Result* Lower dials 776866.056, the double area.

*Step 9* Transfer and multiply by .5.

*Result* Lower dials 388433.028, Area in square feet

### Acreage

To convert the square feet to acreage, perform the following operation.

*Step 10* Divide 388433.028 by 43560, square feet in one acre.

*Result* Upper dials 8.917, Acres

## Closed Traverse and Area — Continued

**Example 2** Figure 6 represents a field, the perimeter of which has been laid out by a transit traverse. Given the values of the bearings and distances which are shown in the illustration, the problem is to close, determine the error of closure, and calculate the area of the field.

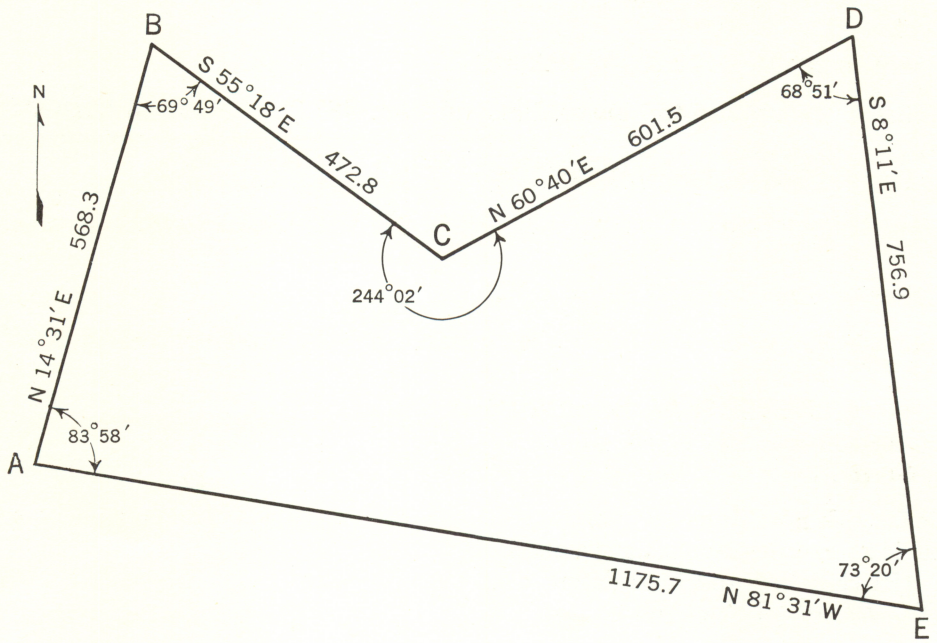



**FIGURE 6**

### LATITUDES AND DEPARTURES

The latitude of each course is calculated by multiplying the distance by the cosine of the bearing and the departure by multiplying the distance by the sine of the bearing, as explained in a previous application.

| COURSE | BEARING | DISTANCE | FUNCTIONS | | COORDINATE DISTANCES | |
|---|---|---|---|---|---|---|
| | | | COS | SIN | COS x DIST | SIN x DIST |
| A-B | N 14° 31′ E | 568.3 | .9680748 | .2506616 | N 550.16 | E 142.45 |
| B-C | S 55° 18′ E | 472.8 | .5692795 | .8221440 | S 269.16 | E 388.71 |
| C-D | N 60° 40′ E | 601.5 | .4898897 | .8717844 | N 294.67 | E 524.38 |
| D-E | S 8° 11′ E | 756.9 | .9898177 | .1423410 | S 749.19 | E 107.74 |
| E-A | N 81° 31′ W | 1175.7 | .1475217 | .9890588 | N 173.44 | W 1162.84 |

Assuming no errors have been made in the measurements of distances and angles, the totals of the north and south latitudes should be the same and the totals of the east and west departures should be the same. In actual practice, however, this condition does not always occur. To check and find the error of the closure, the latitudes and departures are arranged as shown in the following table and balanced.

| POINT | DISTANCE | LATITUDE | | DEPARTURE | | BALANCED LATITUDE | | BALANCED DEPARTURE | |
|---|---|---|---|---|---|---|---|---|---|
| | | N | S | E | W | N | S | E | W |
| A | 568.3 | 173.44 | | 0.00 | 1162.84 | 173.45 | | | 1162.91 |
| B | 472.8 | 550.16 | | 142.45 | | 550.17 | | 142.39 | |
| C | 601.5 | | 269.16 | 388.71 | | | 269.15 | 388.64 | |
| D | 756.9 | 294.67 | | 524.38 | | 294.69 | | 524.29 | |
| E | 1175.7 | 0.00 | 749.19 | 107.74 | | | 749.16 | 107.59 | |
| | 3575.2 | 1018.27 | 1018.35 | 1163.28 | 1162.84 | 1018.31 | 1018.31 | 1162.91 | 1162.91 |
| | | .08 | | | .44 | | | | |

## ERROR OF CLOSURE

$\sqrt{(.08)^2 + (.44)^2} = \sqrt{.20} = .447$

*Step 1* Set .08 on the keyboard. Depress ENTER MULTIPLIER key and hold down. Depress CLEAR MULT.

*Step 2* Set .44 on the keyboard. Depress ENTER MULTIPLIER key and hold down. Depress ACC MULT. *Result* Lower dials .20.

*Step 3* Find the square root of .20 by the Monroe Simplified Method*.

*Result* .447

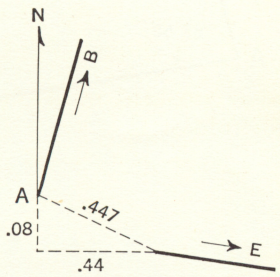


FIGURE 7

Expressed as a ratio to the perimeter the error of closure is .447 in 3575.2 or $\frac{1}{8000}$

## BALANCING

In this example the failure to close is relatively slight and it may be disregarded. However, when it is desired to close the field mathematically, which must be done if the exact area of the field is required, the next step is to balance the survey by adjusting the latitudes and departures so that the totals of the north and south latitudes are equal and the totals of the east and west departures are equal.

### Compass Rule

According to the compass rule: The correction to be applied to the latitude (or departure) of a course is to the total error in latitudes (or departures) as the length of the course is to the length of the traverse (perimeter), thus:

$$\frac{\text{Correction for latitude (departure)}}{\text{Total error in latitudes (departures)}} = \frac{\text{Course}}{\text{Perimeter of field}}$$

*For detailed directions see Monroe Form 1191-S or Form 1228-S.

**Program** Decimals: Keyboard 3 Tab at 9
Upper Dials 8 Transfer Slide 8
Lower Dials 11

### Latitudes

*Step 1* Set .08, error in latitudes, on the keyboard and depress ENTER DIVD. Set 3575.2, perimeter of field, on the keyboard and divide. *Result* Upper dials .00002237, correction factor.

*Step 2* Set the correction factor on the keyboard as 2.237 and register as a constant multiplier by depressing ENTER MULTIPLIER and moving constant multiplier lever down. Multiply by 568.3, distance A-B. Add 173.44, N latitude of A.

*Result* Lower dials 173.45, Corrected N latitude of A

*Step 3* Multiply by 472.8, distance B-C. Add 550.16, N latitude of B.

*Result* Lower dials 550.17, Corrected N latitude of B

*Step 4* Multiply by 756.9, distance D-E. Add 294.67, N latitude of D.

*Result* Lower dials 294.69, Corrected N latitude of D

*Step 5* Multiply negatively by 601.5, distance C-D. Add 269.16, S latitude of C.

*Result* Lower dials 269.15, Corrected S latitude of C

*Step 6* Raise constant multiplier lever up. Multiply negatively by 1175.7, distance E-A. Add 749.19, S latitude of E.

*Result* Lower dials 749.16, Corrected S latitude of E

### Departures

The same procedure is followed for correcting and balancing the departures.

*Step 7* Set .44, error in departures, on the keyboard and depress ENTER DIVD. Set 3575.2, perimeter of field, on the keyboard and divide. *Result* Upper dials .00012307, correction factor for departures.

*Step 8* Set the correction factor on the keyboard as 12.307 and register as a constant multiplier. Multiply by 568.3, distance A-B. Add 1162.84, W departure of A.

*Result* Lower dials 1162.91, Corrected W departure of A

Continue in the same way for correcting the rest of the departures.

## COORDINATES

As north latitudes and east departures are positive and south latitudes and west departures are negative, it is necessary to select the origin so that all points are in the first quadrant and thus all coordinates will have positive signs. The procedure is to have the Y axis pass through the most westerly point and the X axis pass through the most southerly point; in other words, use 0.0 as the E:W coordinate of the most westerly point and 0.0 as the N:S coordinate of the most southerly point. See Figure 8.

| POINT | LATITUDE | | DEPARTURE | |
|---|---|---|---|---|
| | N | S | E | W |
| A | 173.45 | | | 1162.91 |
| B | 550.17 | | 142.39 | |
| C | | 269.15 | 388.64 | |
| D | 294.69 | | 524.29 | |
| E | | 749.16 | 107.59 | |

**Program** Decimals: Keyboard 3 Tab at 4
Upper Dials 3
Lower Dials 6

## N Coordinates

*Step 1* Set 173.45, N latitude of A, the most westerly point, on the keyboard and depress ENTER DIVD. Add 550.17, N latitude of B.

*Result* Lower dials 723.62, N coordinate of B

*Step 2* From the 723.62 in the lower dials subtract 269.15, S latitude of C.

*Result* Lower dials 454.47, N coordinate of C

*Step 3* To the 454.47 in the lower dials add 294.69, N latitude of D.

*Result* Lower dials 749.16, N coordinate of D

## E Coordinates

*Step 4* Clear dials. Add 142.39, E departure of B, and 388.64, E departure of C.

*Result* Lower dials 531.03, E coordinate of C

*Step 5* To the 531.03 in the lower dials add 524.29, E departure of D.

*Result* Lower dials 1055.32, E coordinate of D

*Step 6* To the 1055.32 add 107.59, E departure of E.

*Result* Lower dials 1162.91, E coordinate of E

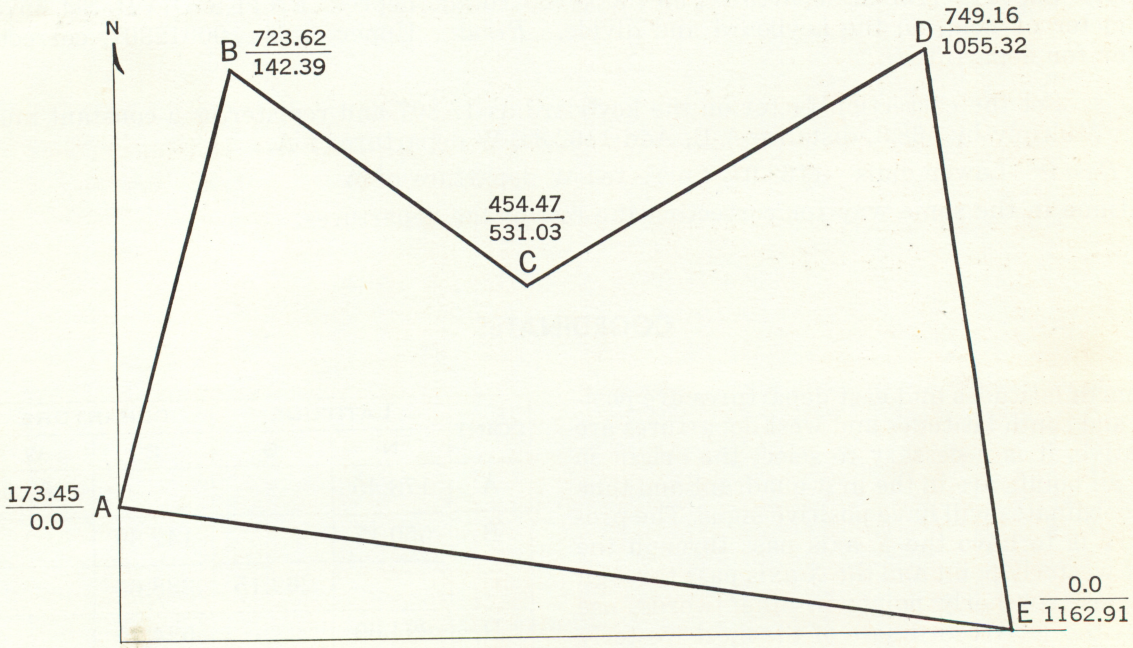



**FIGURE 8**

## AREA

### Coordinate Method

Calculating the area by means of coordinates with a Monroe is greatly simplified by arranging the values as shown below. In the table, the solid lines connecting the coordinates indicate those multiplied positively and accumulated; the broken lines indicate the coordinates multiplied negatively.

| POINT | A | B | C | D | E | A |
|---|---|---|---|---|---|---|
| NORTH COORDINATE | 173.45 | 723.62 | 454.47 | 749.16 | 0.0 | 173.45 |
| EAST COORDINATE | 0.0 | 142.39 | 531.03 | 1055.32 | 1162.91 | 0.0 |

**Program** Decimals: Keyboard 3 Tab at 4
Upper Dials 3 AUTO KB CLEAR to Right
Lower Dials 6

The following steps are performed consecutively without clearing the machine so that all results of positive and negative multiplications are accumulated.

*Step 1* Multiply 173.45, N coordinate of A, by 142.39, E coordinate of B. Multiply negatively 142.39 by 454.47, N coordinate of C.

*Step 2* Multiply accumulatively 454.47 by 1055.32, E coordinate of D.

Since N coordinate of E is 0.0 no multiplication is required.

*Step 3* Multiply negatively 173.45, N coordinate of A, by 1162.91, E coordinate of E.

*Step 4* Multiply accumulatively 1162.91 by 749.16, N coordinate of D. Multiply negatively 749.16 by 531.03, E coordinate of C.

*Step 5* Multiply accumulatively 531.03 by 723.62, N coordinate of B.

Since E coordinate of A is 0.0 no multiplication is required. At this point the result in the lower dials is 1095533.252, double area.

*Step 6* Divide amount in lower dials by 2.

*Result* Upper dials 547766.626, Area

## Closed Traverse and Area — Continued

**Example 3** The closed traverse represented by Figure 9 introduces a somewhat complex condition because the field is an eight-sided figure with the closing side intersecting one of the other sides. It is necessary to find the coordinates of the intersecting point in order to calculate the area.

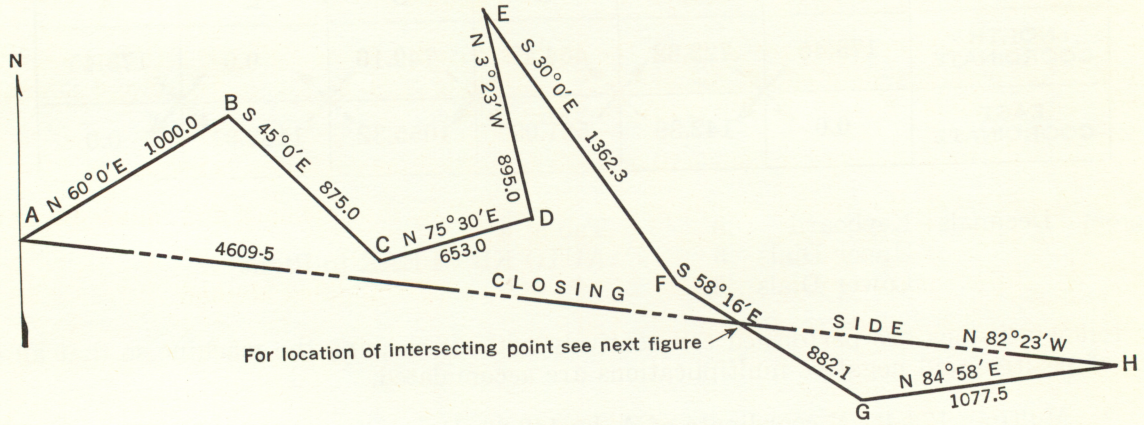



FIGURE 9

## COORDINATE DISTANCES AND COORDINATES

First the latitude of each course is calculated by multiplying the distance by the cosine of the bearing and the departure by multiplying the distance by the sine of the bearing, as previously described. In this instance 1000 is arbitrarily added to the N:S coordinate of the origin (point A) in order to have uniform, positive signs for all the N:S coordinates. The values are given in the table.

| SIDE | BEARING | DISTANCE | COORDINATE DISTANCES | | COORDINATES | | POINT |
|---|---|---|---|---|---|---|---|
| | | | LATITUDE | DEPARTURE | LATITUDE | DEPARTURE | |
| | | | | | N 1000.0 | 0.0 | A |
| A-B | N 60° 0′ E | 1000.0 | N 500.0 | E 866.0 | N 1500.0 | E 866.0 | B |
| B-C | S 45° 0′ E | 875.0 | S 618.7 | E 618.7 | N 881.3 | E 1484.7 | C |
| C-D | N 75° 30′ E | 653.0 | N 163.5 | E 632.2 | N 1044.8 | E 2116.9 | D |
| D-E | N 3° 23′ W | 895.0 | N 893.4 | W 52.8 | N 1938.2 | E 2064.1 | E |
| E-F | S 30° 0′ E | 1362.3 | S 1179.8 | E 681.2 | N 758.4 | E 2745.3 | F |
| F-G | S 58° 16′ E | 882.1 | S 463.9 | E 750.2 | N 294.5 | E 3495.5 | G |
| G-H | N 84° 58′ E | 1077.5 | N 94.5 | E 1073.4 | N 389.0 | E 4568.9 | H |
| H-A | N 82° 23′ W | 4609.5 | | | | | |
| | | | | | N 598.2 | E 3004.4 | P |

**Program** Decimals: Keyboard 5 Tab at 6
Upper Dials 5
Lower Dials 10

## CLOSING SIDE

### Length

| | | |
|---|---|---|
| Coordinates given end | N:S 1000.0 N | E:W 0.0 |
| Coordinates calculated end | N:S 389.0 N | E:W 4568.9 E |
| | 611.0 | 4568.9 |

*Step 1* Set 1000.0, N coordinate of A, on the keyboard and depress ENTER DIVD to register in lower dials correctly pointed off. Set 389.0, N coordinate of H, on the keyboard and subtract. *Result* Lower dials 611.00, N:S coordinate difference.

As E:W coordinate difference is known to be 4568.9, no machine operation is necessary.

*Step 2* Set 611.0 on the keyboard and square. Set 4568.9 on the keyboard and square by using ACC MULT key. *Result* Lower dials 21248168.21.

*Step 3* Find the square root of 21248168.21 by the Monroe Simplified Method.
*Result* 4609.5, Length of closing side A-H

### Bearing

$$\text{Tan} = \frac{\text{E:W Coordinate Difference}}{\text{N:S Coordinate Difference}}$$

*Step 4* Divide 4568.9 by 611.0.
*Result* Upper dials 7.47774, Tan of 82° 23′

Thus the closing side of A-H has a distance of 4609.5 and a bearing of N 82° 23′ W.

## INTERSECTING POINT

Next the point where the closing side H-A intersects side F-G must be found to determine its coordinates which are required for computing the area. This is graphically shown in Figure 10.

Tan 58° 16′ = 1.61703
Tan 82° 23′ = 7.47774

$$\text{Tan } 58° 16' = \frac{S}{R}$$

$$1.61703 = \frac{S}{R}$$

$$S = 1.61703R$$

$$\text{Tan } 82° 23' = \frac{S + 1073.3}{R - 94.5}$$

$$7.47774 = \frac{1.61703R + 1073.3}{R - 94.5}$$

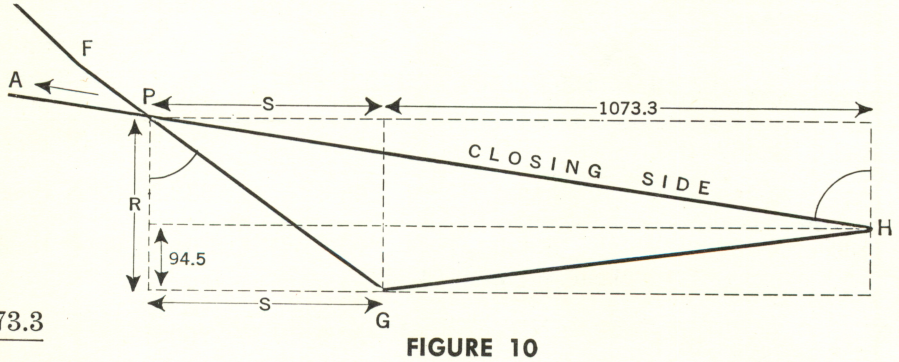


FIGURE 10

Solving for R and S R = 303.7 S = 491.1

*Step 1* Add 294.5, N coordinate of G, and 303.7, R.
*Result* Lower dials 598.2, N coordinate of P

*Step 2* From 3495.5, E coordinate of G, subtract 491.1, S.

*Result* Lower dials 3004.4, E coordinate of P

The coordinates of the intersecting point P are N 598.2 and E 3004.4.

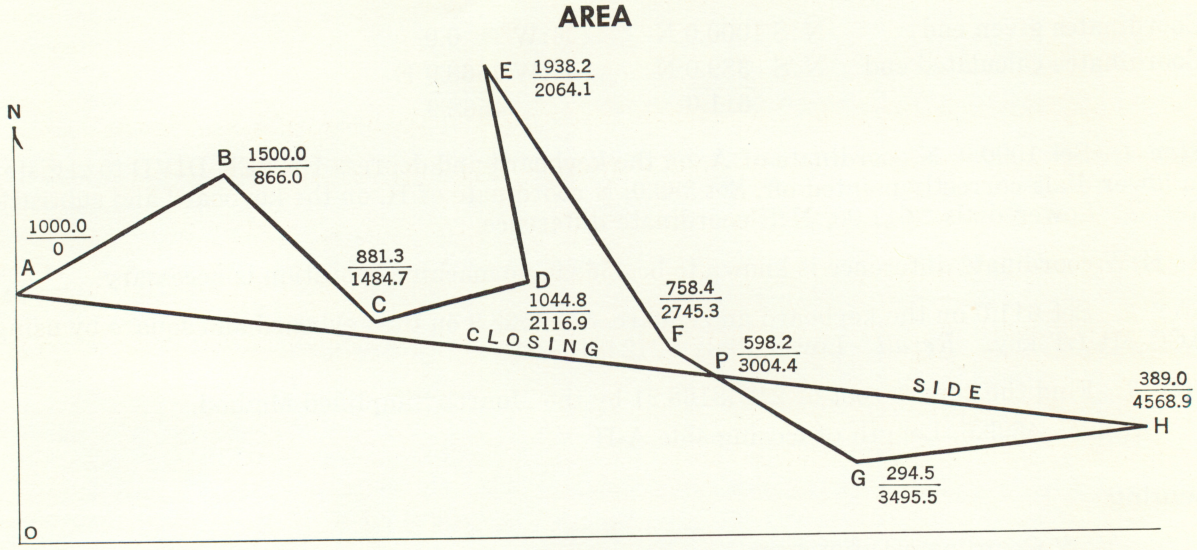



**FIGURE 11**

The closing side H-A intersects the side F-G (see Figures 9 and 10), thereby actually forming two figures. However, as the problem is to find the area of the entire traverse it is not necessary to compute the areas of the two figures separately and then add them as the result can be secured with the Monroe Calculator in one simple, continuous operation. The coordinate method is used. The coordinates of all points are given in Figure 11. For convenience these values are arranged as follows.

| POINT | A | B | C | D | E | F | P | H | G | P | A |
|---|---|---|---|---|---|---|---|---|---|---|---|
| NORTH COORDINATE | 1000.0 | 1500.0 | 881.3 | 1044.8 | 1938.2 | 758.4 | 598.2 | 389.0 | 294.5 | 598.2 | 1000.0 |
| EAST COORDINATE | 0.0 | 866.0 | 1484.7 | 2116.9 | 2064.1 | 2745.3 | 3004.4 | 4568.9 | 3495.5 | 3004.4 | 0.0 |

**Program** Decimals: Keyboard 2 — Tab at 3
Upper Dials 2 — AUTO KB CLEAR to Right
Lower Dials 4

*Step 1* Multiply 1000.0, N coordinate of A, by 866.0, E coordinate of B. Multiply negatively 866.0 by 881.3, N coordinate of C.

*Step 2* Multiply accumulatively 881.3 by 2116.9, E coordinate of D. Multiply negatively 2116.9 by 1938.2, N coordinate of E.

*Step 3* Multiply accumulatively 1938.2 by 2745.3, E coordinate of F. Multiply negatively 2745.3 by 598.2, N coordinate of P.

*Step 4* Multiply accumulatively 598.2 by 4568.9, E coordinate of H. Multiply negatively 4568.9 by 294.5, N coordinate of G.

*Step 5* Multiply accumulatively 294.5 by 3004.4, E coordinate of P. Multiply negatively 3004.4 by 1000.0, N coordinate of A.

At this point the lower dials read 812115.32. Do not clear machine.

As the E coordinate of A is 0.0 no multiplication is required.

*Step 6* Multiply negatively 598.2 by 3495.5. Multiply accumulatively 3495.5 by 389.0.

*Step 7* Multiply negatively 389.0 by 3004.4. Multiply accumulatively 3004.4 by 758.4.

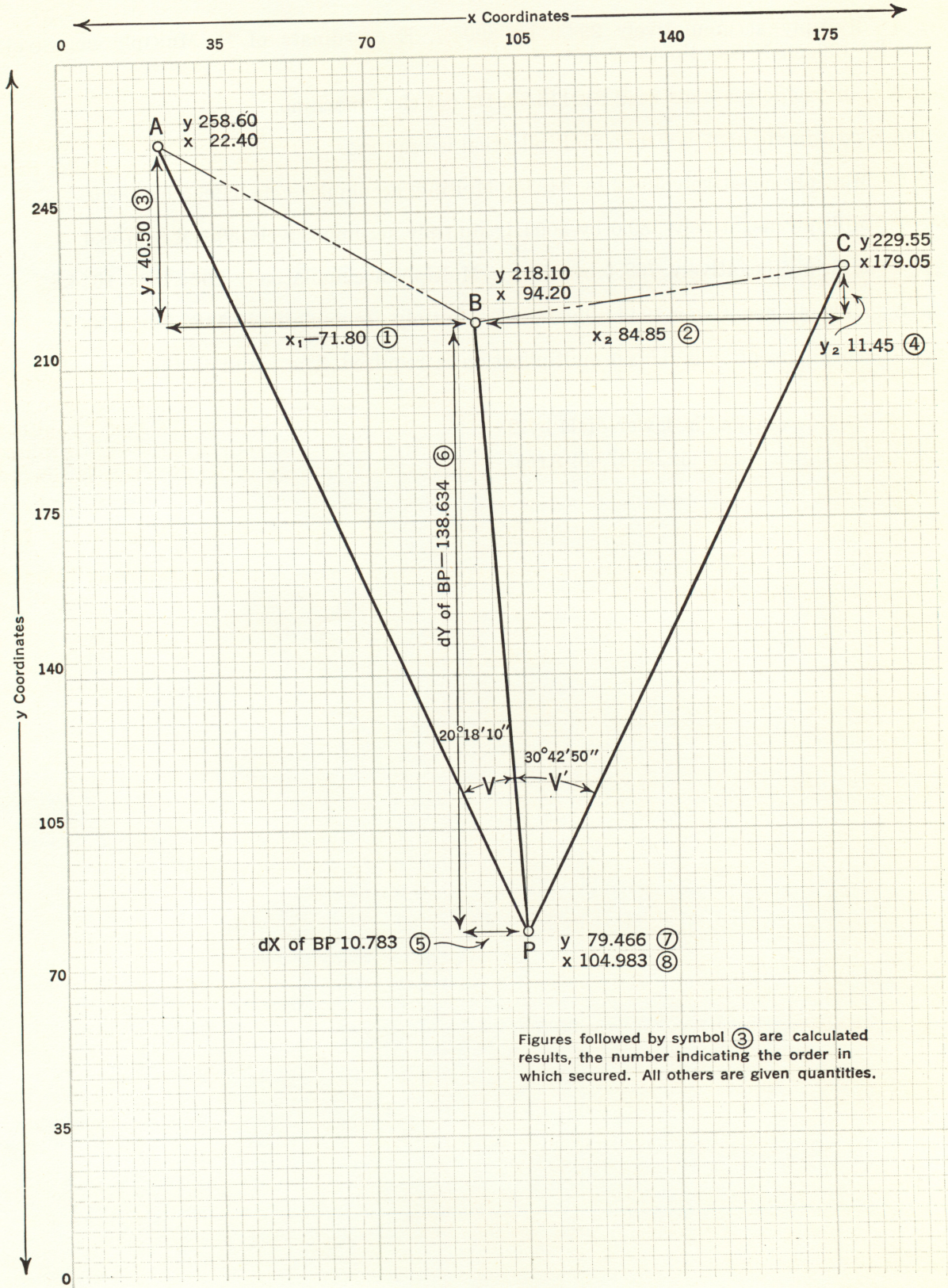
*Step 8* Multiply negatively 758.4 by 2064.1. Multiply accumulatively 2064.1 by 1044.8.

*Step 9* Multiply negatively 1044.8 by 1484.7. Multiply accumulatively 1484.7 by 1500.0.

Since E coordinate of A is 0.0 no multiplication is required. The result in the lower dials is 2457675.76, double area.

*Step 10* Divide amount in lower dials by 2.

*Result* Upper dials 1228837.88, Area




Figures followed by symbol ③ are calculated results, the number indicating the order in which secured. All others are given quantities.

# Three-Point Problem

## ANALYTIC SOLUTION

***Example*** Find the coordinates $P_x$ and $P_y$ in the figure illustrated.

The solution of the Three-Point Problem by this method is based upon taking the analytic geometry equations for the circle through ABP and the circle through PBC, and solving them as simultaneous quadratic equations for the dX and dY of BP.

**Given**

| | | | | | |
|---|---|---|---|---|---|
| Coordinates | $A_x$ | 22.40 | $B_x$ | 94.20 | $C_x$ 179.05 |
| | $A_y$ | 258.60 | $B_y$ | 218.10 | $C_y$ 229.55 |
| Angle V | | 20° 18′ 10″ | | | |
| | $V^1$ | 30° 42′ 50″ | | | |

dX represents the difference in the X coordinates of a line and dY represents the difference in the Y coordinates. The values are found by subtracting the coordinates of the beginning of the line from the coordinates of the end of the line. Thus dX of BA = x of A − x of B; similar for dY. The smaller coordinate is subtracted from the greater; the sign of the result is minus if the direction of the line is toward the smaller coordinate or plus if the direction is toward the larger coordinate.

Let $x_1$ and $y_1$ represent the dX and dY of BA; and $x_2$ and $y_2$ represent the dX and dY of BC.

Each step in the solution is numbered to correspond with the steps in the detailed description of the Monroe machine method.

### Working Data

**1** dX of BA = $x_1$ = −71.80 dX of BC = $x_2$ = 84.85
dY of BA = $y_1$ = 40.50 dY of BC = $y_2$ = 11.45

**2** $$K = \frac{(x_1 - y_1 \operatorname{Cot} V) - (x_2 + y_2 \operatorname{Cot} V^1)}{(y_2 - x_2 \operatorname{Cot} V^1) - (y_1 + x_1 \operatorname{Cot} V)}$$

$$= \frac{[-71.8 - (40.5 \times 2.70295)] - [84.85 + (11.45 \times 1.68326)]}{[11.45 - (84.85 \times 1.68326)] - [40.5 + (-71.8 \times 2.70295)]}$$

$$= \frac{(-71.8 - 109.46948) - (84.85 + 19.27333)}{(11.45 - 142.82461) - (40.5 - 194.07181)}$$

$$= \frac{(-181.26948) - (104.12333)}{(-131.37461) - (-153.57181)} = \frac{-285.39281}{22.19720}$$

$$K = -12.85715$$

**3** $$\text{dX of BP} = \frac{K(y_1 + x_1 \operatorname{Cot} V) + (x_1 - y_1 \operatorname{Cot} V)}{K^2 + 1}$$

$$= \frac{-12.85715\ (-153.57181) + (-181.26948)}{(-12.85715)^2 + 1}$$

$$= \frac{1974.49580 - 181.26948}{166.30631} = \frac{1793.22632}{166.30631}$$

$$\text{dX of BP} = 10.78267$$

**4 To Prove**

$$\text{dX of BP} = \frac{K\,(y_2 - x_2 \operatorname{Cot} V^1) + (x_2 + y_2 \operatorname{Cot} V^1)}{K^2 + 1}$$

$$= \frac{-12.85715\ (-131.37461) + 104.12333}{166.30631}$$

$$= \frac{1793.22640}{166.30631}$$

$$\text{dX of BP} = 10.78267$$

**5** $\text{dY of BP} = K\text{dX of BP}$

$= -12.85715 \times 10.78267$

$\text{dY of BP} = -\ 138.63441$

**6** $P_y - B_y = \text{dY of BP}$

$P_y - 218.1 = -138.63441$

$P_y = 79.46559$, y coordinate of point P

**7** $P_x - B_x = \text{dX of BP}$

$P_x - 94.2 = 10.78267$

$P_x = 104.98267$, x coordinate of point P

## Monroe Machine Method

| **Program** | Decimals: | Keyboard | 5 | Tab at 6 |
|---|---|---|---|---|
| | | Upper Dials | 5 | Transfer Slide 5 |
| | | Lower Dials | 10 | AUTO KB CLEAR to Right |

All additions and subtractions should be performed with the carriage in the sixth position.

*Step 1* To find the dX and dY of BA and BC, subtract the coordinates of the beginning of the line from the coordinates of the end of the line. The smaller coordinate should be subtracted from the greater; the sign of the result (dX or dY) is minus if the direction of the line is toward the smaller coordinate or plus if the direction is toward the greater.

| | | |
|---|---|---|
| From 94.2, $B_x$, subtract 22.4, $A_x$ | Result in lower dials is | $-71.80$, $x_1$ ① |
| From 179.05, $C_x$, subtract 94.2, $B_x$ | Result in lower dials is | 84.85, $x_2$ ② |
| From 258.6, $A_y$, subtract 218.1, $B_y$ | Result in lower dials is | 40.50, $y_1$ ③ |
| From 229.55, $C_y$, subtract 218.1, $B_y$ | Result in lower dials is | 11.45, $y_2$ ④ |

*Step 2* From tables, cot of angle V, 20° 18′ 10″, is 2.70295; cot of angle $V^1$, 30° 42′ 50″, is 1.68326.

It will be noted that the expressions in the formula for finding K are repeated in the formulas that follow it. Therefore, for convenience,

Let $(x_1 - y_1 \operatorname{Cot} V\ ) = M$
$(x_2 + y_2 \operatorname{Cot} V^1) = N$
$(y_2 - x_2 \operatorname{Cot} V^1) = L$
$(y_1 + x_1 \operatorname{Cot} V\ ) = R$

In solving these expressions, careful attention must be given to the signs of the operations. It

is suggested that the results of the expressions should be noted in their proper places in the formulas as they are secured.

Find $(x_1 - y_1 \text{ Cot } V)$

Set 71.80, $x_1$, on the keyboard and depress ENTER DIVD to register in lower dials. Multiply accumulatively 40.5, $y_1$, by 2.70295, Cot V. The result in the lower dials is −181.26948, value of M.

Find $(y_1 + x_1 \text{ Cot } V)$

Multiply 2.70295, Cot V, by 71.80, $x_1$. Subtract 40.5, $y_1$. Result in lower dials is −153.57181, value of R.

Find $(x_2 + y_2 \text{ Cot } V^1)$

Set 84.85, $x_2$, on the keyboard and depress ENTER DIVD to register in lower dials. Multiply accumulatively 11.45, $y_2$, by 1.68326, Cot $V^1$. Result in lower dials is 104.12333, value of N.

Find $(y_2 - x_2 \text{ Cot } V^1)$

Multiply 1.68326, Cot $V^1$, by 84.85, $x_2$. Subtract 11.45, $y_2$. The result in the lower dials is −131.37461, value of L.

Find $K = \dfrac{M - N}{L - R}$

Move AUTO KB CLEAR to left. From 153.57181, R, subtract 131.37461, L. Result in lower dials is 22.19720, L − R, which is noted. Clear dials.

Add 181.26948, M, and 104.12333, N. Result in lower dials is −285.39281, M − N. Divide by 22.19720, L − R. Result in upper dials is −12.85715, value of K.

*Step 3* Find $K^2 + 1$.

Set 1 on the keyboard and depress ENTER DIVD to register in lower dials. Square accumulatively 12.85715, K. Result in lower dials is 166.30631, $K^2 + 1$.

Find dX of BP $= \dfrac{KR + M}{K^2 + 1}$

Multiply 12.85715, K, by 153.57181, R. Subtract 181.26948, M. Divide by 166.30631, $K^2 + 1$. Result in upper dials is 10.78267 ⑤, dX of BP, which is noted.

*Step 4* Prove dX of BP $= \dfrac{KL + N}{K^2 + 1}$

Multiply 12.85715, K, by 131.37461, L. Add 104.12333, N. Divide by 166.30631, $K^2 + 1$. Result in upper dials is 10.78267 ⑤, which checks with result of Step 3 for proof.

*Step 5* Find dY of BP = KdX

Multiply 12.85715, K, by 10.78267, dX. Result in lower dials is −138.63441, dY of BP⑥.

*Step 6* Find $P_y - B_y$ = dY of BP

Subtract 138.63441, dY, from 218.10, $B_y$.

*Result* Lower dials 79.466, $P_y$ ⑦

*Step 7* Find $P_x - B_x$ = dX of BP

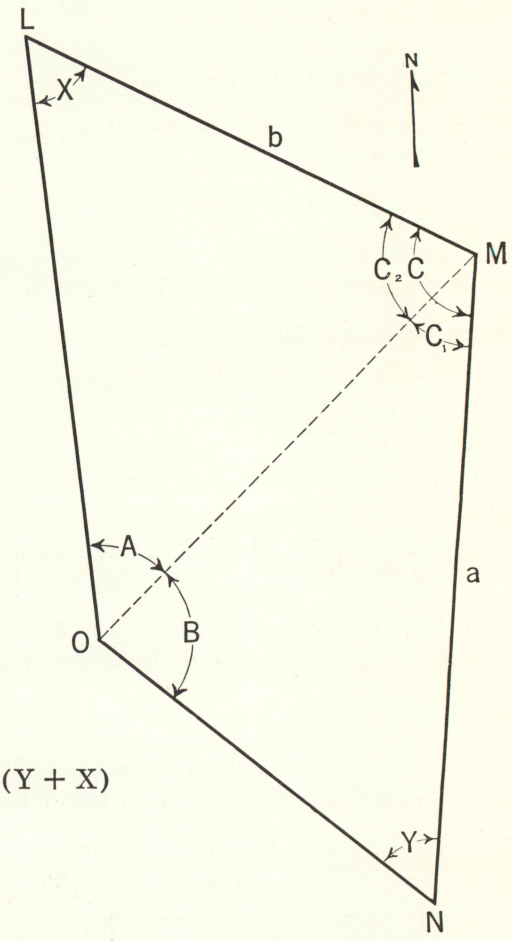
Clear dials. Add 10.78267, dX, and 94.20, $B_x$.

*Result* Lower dials 104.98267, $P_x$ ⑧

## Three-Point Problem — Continued

### TRIGONOMETRIC SOLUTION I

**Example** Find the lengths of the sides $\overline{LO}$ and $\overline{NO}$ and the coordinates of point O.




**Given**

| | |
|---|---|
| Angle A | 54° 39′ 20″ |
| Angle B | 81° 9′ 50″ |
| Angle C | 113° 0′ 7″ |
| Side a | 9106.37 |
| Side b | 6605.59 |
| Bearing $\overline{NO}$ | N 44° 21′ 5″ W |
| Bearing $\overline{LO}$ | S 0° 10′ 15″ E |
| Coordinates of L | N 4023909.83 |
| | E 3318630.93 |
| Coordinates of N | N 4011663.00 |
| | E 3323487.40 |

**Formulas**

$$Y + X = 360° - A - B - C$$

$$\text{Tan } \tfrac{1}{2}(Y - X) = \frac{b \text{ Sin } B - a \text{ Sin } A}{b \text{ Sin } B + a \text{ Sin } A} \times \text{Tan } \tfrac{1}{2}(Y + X)$$

**Working Data**

1 $Y + X = 360° - A - B - C = 111° 10′ 43″$
$\tfrac{1}{2}(Y + X) = 55° 35′ 22″$

2 $\text{Sin } A = .8156891$
$\text{Sin } B = .9881318$
$\text{Tan } \tfrac{1}{2}(Y + X) = 1.4598861$

3 $b \text{ Sin } B + a \text{ Sin } A = 13955.16$

4 $\dfrac{b \text{ Sin } B - a \text{ Sin } A}{b \text{ Sin } B + a \text{ Sin } A} \times \text{Tan } \tfrac{1}{2}(Y + X) = \text{Tan } \tfrac{1}{2}(Y - X) = .0942321$

5 $\tfrac{1}{2}(Y + X) = 55° 35′ 22″$

6 $\tfrac{1}{2}(Y - X) = -5° 23′ 0″$

7 $Y = 50° 12′ 22″$

8 $X = (Y + X) - Y = 60° 58′ 21″$

**9** $C_1 = 180° - B - Y = 48° 37' 48''$
$C_2 = 180° - A - X = 64° 22' 19''$

**10** $\text{Sin } C_1 = .7504572$
$\text{Sin } C_2 = .9016208$

**11-A** $\text{Cos bearing } \overline{NO} = .7150660$
$\text{Sin bearing } \overline{NO} = .6990569$

**11-B** $\overline{NO} = \dfrac{a \text{ Sin } C_1}{\text{Sin } B} = 6916.021$

| | | | | | |
|---|---|---|---|---|---|
| **11-C** | $\overline{NO} \times \text{Cos bearing } \overline{NO} = N$ | 4945.41 (N:S) | $\overline{NO} \times \text{Sin bearing } \overline{NO} = W$ | 4834.69 (E:W) |
| **11-D** | Coordinate of N | = N 4011663.00 (N:S) | Coordinate of N | = E 3323487.40 (E:W) |
| **11-E** | Coordinate of O | = N 4016608.41 (N:S) | Coordinate of O | = E 3318652.71 (E:W) |

**12-A** $\text{Cos bearing } \overline{LO} = .9999956$
$\text{Sin bearing } \overline{LO} = .0029816$

**12-B** $\overline{LO} = \dfrac{b \text{ Sin } C_2}{\text{Sin } A} = 7301.479$

| | | | | | |
|---|---|---|---|---|---|
| **12-C** | $\overline{LO} \times \text{Cos bearing } \overline{LO} = S$ | 7301.44 (N:S) | $\overline{LO} \times \text{Sin bearing } \overline{LO} = E$ | 21.77 (E:W) |
| **12-D** | Coordinate of L | = N 4023909.83 (N:S) | Coordinate of L | = E 3318630.93 (E:W) |
| **12-E** | Coordinate of O | = N 4016608.39 (N:S) | Coordinate of O | = E 3318652.70 (E:W) |

Note 12-E and 11-E check

## Monroe Machine Method

**Program** Decimals: Keyboard 3 - 7
Upper Dials 3 - 7
Lower Dials 3 - 10

Set all values with three decimal places or less at the third decimal of the keyboard; set all others at the seventh decimal.

Each step of the machine method is numbered to correspond with the numbered line of the working data above.

*Step 1* From 360° subtract 54° 39′ 20″, 81° 9′ 50″, and 113° 0′ 7″. (Use Monroe method previously described.) The result is 111° 10′ 43″, ½ of which is 55° 35′ 22″.

*Step 2* Values of sin A, sin B, and tan ½ (Y + X) are taken from tables of natural functions. Interpolations are made on the Monroe machine by following the method previously explained.

*Step 3* Multiply .9881318, sin B, by 6605.59, b. Multiply accumulatively .8156891, sin A, by 9106.37, a. The result in the lower dials, 13955.16, should be noted.

*Step 4* Multiply negatively twice .8156891, sin A, by 9106.37, a. The result in the lower dials is −900.7732130. Multiply 900.7732130 by 1.4598861. Divide by 13955.16. The result in the upper dials is .09423226, ½ (Y − X).

*Step 5* ½ (Y + X) was secured in Step 1.

*Step 6* From tables, .09423226 is found to be the tan of 5° 23′ 00″.

*Step 7* Adding the equations cancels the X's leaving 2 × ½ Y or Y which equals the sum of the angles, 50° 12′ 22″.

*Step 8* From 111° 10′ 43″, Y + X, subtract 50° 12′ 22″, Y. Result in the lower dials is 60° 58′ 21″.

*Step 9* From 180° subtract 81° 9′ 50″ and 50° 12′ 22″. Result is 48° 37′ 48″, $C_1$. From 180° subtract 54° 39′ 20″ and 60° 58′ 21″. Result is 64° 22′ 19″, $C_2$.

*Step 10* Values for sin $C_1$ and sin $C_2$ are found in tables.

*Step 11-A* Values for cos bearing $\overline{NO}$ and sin bearing $\overline{NO}$ are found in tables.

*Step 11-B* Multiply .7504572, sin $C_1$, by 9106.37, a. Divide by .9881318, sin B.

*Result* Upper dials 6916.021, $\overline{NO}$

*Step 11-C* Copy 6916.021 from the upper dials to keyboard and enter as a constant multiplier. Multiply by .7150660, cos bearing $\overline{NO}$. Result in lower dials is 4945.41. Release constant lever and multiply by .6990569, sin bearing $\overline{NO}$. Result in lower dials is 4834.69.

*Step 11-D* Clear dials. Add 4011663.00 and 4945.41.

*Result* Lower dials 4016608.41, N coordinate of O

*Step 11-E* Clear dials. Add 3323487.40 and subtract 4834.69.

*Result* Lower dials 3318652.71, E coordinate of O

*Step 12-A* The values for cos and sin of bearing $\overline{LO}$ are found in tables.

*Step 12-B* Multiply .9016208, sin $C_2$, by 6605.59, b. Divide by .8156891, sin A.

*Result* Upper dials 7301.479, $\overline{LO}$

*Steps 12-C and 12-D* Perform in the same way as Steps 11-C and 11-D. The signs of the distances are obvious to the engineer.

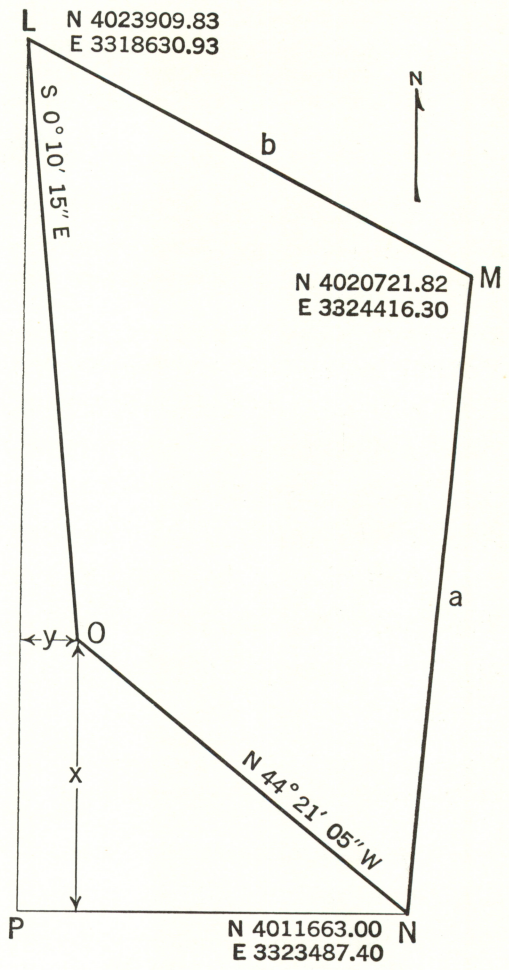
## Three-Point Problem — Continued

### TRIGONOMETRIC SOLUTION II

**Example** Find the coordinates of point O.

**Given**

| | | |
|---|---|---|
| Coordinates of L | N 4023909.83 | |
| | E 3318630.93 | |
| Coordinates of N | N 4011663.00 | |
| | E 3323487.40 | |
| Bearing $\overline{LO}$ | S | 0° 10′ 15″ E |
| Bearing $\overline{NO}$ | N | 44° 21′ 05″ W |




**Formulas**

$$\text{Tan bearing } \overline{LO} = \frac{y}{LP - x}$$

$$\text{Tan bearing } \overline{NO} = \frac{NP - y}{x}$$

**Working Data**

1 N:S coordinate of L N 4023909.83
N:S coordinate of N N 4011663.00
LP 12246.83

2 E:W coordinate of L E 3318630.93
E:W coordinate of N E 3323487.40
NP 4856.47

3 Tan bearing $\overline{LO}$ = Tan 0° 10′ 15″ = .0029816

$$\frac{y}{LP - x} = .0029816$$

$$\frac{y}{12246.83 - x} = .0029816$$

$$y = .0029816\ (12246.83 - x) = 36.515148 - .0029816x$$

4 Tan bearing $\overline{NO}$ = Tan 44° 21′ 05″ = .9776117

$$\frac{NP - y}{x} = .9776117$$

$$.9776117 = \frac{4856.47 - (36.515148 - .0029816x)}{x}$$

$$.9776117x = 4856.47 - (36.515148 - .0029816x)$$
$$.9746301x = 4819.954852$$
$$x = 4945.42$$

**5** Substituting value of x
$y = 36.51548 - .0029816\ (4945.42)$
$y = 21.77$

**6**

| | | |
|---|---|---|
| Coordinate point L | | E 3318630.93 |
| Coordinate point N | N 4011663.00 | |
| Values of x and y | N 4945.42 | E 21.77 |
| Coordinates point O | N 4016608.42 | E 3318652.70 |

## Monroe Machine Method

**Program** Decimals: Keyboard 3 - 7 Tab at 4
Upper Dials 3 - 7
Lower Dials 3 - 10

All values with three decimal places or less are set at the third decimal of the keyboard; all others are set at the seventh decimal.

As in the previous problem, the steps are numbered to correspond with the numbered lines of the working data.

*Step 1* From 4023909.83, N:S coordinate of L, subtract 4011663.00, N:S coordinate of N. Result in the lower dials is 12246.83, LP.

*Step 2* From 3323487.40, E:W coordinate of N, subtract 3318630.93, E:W coordinate of L. Result in the lower dials is 4856.47, NP.

*Step 3* Tan of bearing 0° 10′ 15″ is found in tables to be .0029816. Multiply .0029816 by 12246.83, LP. The result in the lower dials is 36.5151483, which is substituted in the equation.

*Step 4* Tan of bearing 44° 21′ 05″ is found in tables to be .9776117, which is substituted in the equation.

From .9776117 subtract .0029816. Result in lower dials is .9746301.

From 4856.47 subtract 36.5151483. Result in lower dials is 4819.9548517.

Divide the 4819.9548517 by .9746301. Result in upper dials is 4945.42, x.

*Step 5* Substitute value of x in the equation for y. Enter dividend 36.5151483. Multiply negatively .0029816 by 4945.42. Result in the lower dials is 21.77, y.

*Step 6* Add 4011663.00, N:S coordinate of point N, and 4945.42, value of x.

*Result* Lower dials 4016608.42, N coordinate of O

Clear dials. Add 3318630.93, E:W coordinate of L, and 21.77, value of y.

*Result* Lower dials 3318652.70, E coordinate of O

# End Areas and Cuts and Fills

Earth work volumes are usually computed by the average end area method. The areas of the cross section of the cuts and fills are figured by coordinates.

**Example** Figure 12 is a sketch of an end area with necessary data for calculating the end areas of the cuts and fill.

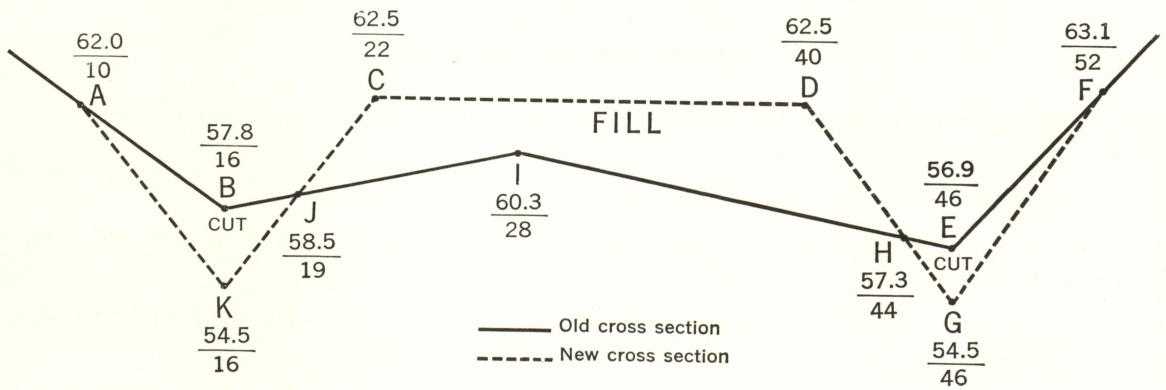



**FIGURE 12**

**Program** Decimals: Keyboard 2 Tab at 3
Upper Dials 2 Transfer Slide 2
Lower Dials 4 AUTO KB CLEAR to Right

## END AREAS

### Cut A B J K

Starting at the extreme right-hand point and moving clockwise, the coordinates are arranged in the table. The solid lines connecting the coordinates indicate those multiplied positively and accumulated; the broken lines indicate those multiplied negatively.

| POINT | J | K | A | B | J |
|---|---|---|---|---|---|
| ELEVATION | 58.5 | 54.5 | 62.0 | 57.8 | 58.5 |
| DISTANCE | 19. | 16. | 10. | 16. | 19. |

*Step 1* Set 58.5 on the keyboard and multiply accumulatively by 16. Multiply negatively 16 by 62.0.

*Step 2* Multiply accumulatively 62.0 by 16. Multiply negatively 16 by 58.5.

*Step 3* Clear keyboard. Set 19 on keyboard and multiply accumulatively by 57.8. Multiply negatively 57.8 by 10.

*Step 4* Multiply accumulatively 10 by 54.5. Multiply negatively 54.5 by 19. *Result* Lower dials 29.70, Double end area.

*Step 5* Clear keyboard. Transfer and multiply by .5.

*Result* Lower dials 14.85, End area of cut A B J K

### Cut H E F G

| POINT | F | G | H | E | F |
|---|---|---|---|---|---|
| ELEVATION | 63.1 | 54.5 | 57.3 | 56.9 | 63.1 |
| DISTANCE | 52. | 46. | 44. | 46. | 52. |

*Step 1* Set 63.1 on the keyboard and multiply accumulatively by 46. Multiply negatively 46 by 57.3.

*Step 2* Multiply accumulatively 57.3 by 46. Multiply negatively 46 by 63.1.

*Step 3* Change keyboard set-up to 52 and multiply accumulatively by 56.9. Multiply negatively 56.9 by 44.

*Step 4* Multiply accumulatively 44 by 54.5. Multiply negatively 54.5 by 52. *Result* Lower dials 19.20, Double end area.

*Step 5* Clear keyboard. Transfer and multiply by .5.

*Result* Lower dials 9.60, End area H E F G

### Fill C D H I J

The same method is followed for figuring the fill, first arranging coordinates of each point.

| POINT | H | I | J | C | D | H |
|---|---|---|---|---|---|---|
| ELEVATION | 57.3 | 60.3 | 58.5 | 62.5 | 62.5 | 57.3 |
| DISTANCE | 44. | 28. | 19. | 22. | 40. | 44. |

*Step 1* Set 57.3 on the keyboard and multiply accumulatively by 28. Multiply negatively 28 by 58.5.

*Step 2* Multiply accumulatively 58.5 by 22. Multiply negatively 22 by 62.5.

*Step 3* Multiply accumulatively 62.5 by 44.

*Step 4* Change keyboard set-up to 57.3 and multiply negatively by 40. Multiply accumulatively 40 by 62.5.

*Step 5* Multiply negatively 62.5 by 19. Multiply accumulatively 19 by 60.3.

*Step 6* Multiply negatively 60.3 by 44. *Result* Lower dials 141.40, Double end area.

*Step 7* Transfer and multiply by .5.

*Result* Lower dials 70.70, End area C D H I J

## YARDAGE

A cross section sketch is made for each station and by the Monroe method just described the end areas of both cuts and fills are figured. A table for a series of stations may be somewhat like the one below from which the cubic yards of cuts and fills are calculated.

| STATION | DISTANCE BETWEEN | SQUARE FEET | | | | CUBIC YARDS | |
|---|---|---|---|---|---|---|---|
| | | CUT | TOTAL CUT | FILL | TOTAL FILL | CUT | FILL |
| 147:30 | | 12 | | 38 | | | |
| 147:76 | .46 | 13 | 25 | 37 | 75 | 21 | 64 |
| 148:30 | .56 | 12 | 25 | 16 | 53 | 26 | 55 |
| 149:00 | .70 | 21 | 33 | 5 | 21 | 43 | 27 |
| 150:00 | 1.00 | 15 | 36 | 9 | 14 | 67 | 26 |
| 151:00 | 1.00 | 17 | 32 | 8 | 17 | 59 | 31 |
| 152:00 | 1.00 | 29 | 46 | 0 | 8 | 85 | 15 |
| 153:00 | 1.00 | 0 | 29 | 17 | 17 | 54 | 31 |

**Formula**

$$\frac{\dfrac{\text{Total end areas of two adjacent stations}}{2} \times \text{Distance between stations}}{27} = \text{Yardage}$$

or

$$\frac{\text{Total end areas} \times \text{Distance}}{54} = \text{Yardage}$$

**Program** Decimals: Keyboard 2 Upper Dials 2 Lower Dials 4 Tabs at 3 and 5

### Cuts

*Step 1* Set 25, total of the two cuts (12 + 13) on the keyboard. Multiply by 46.

*Step 2* Set 54 on the keyboard and divide.

*Result* Upper dials 21, Cubic yards of cut between first two stations

Continue same procedure for figuring cuts between all other stations.

### Fills

*Step 1* Set 75, total of the fills (38 + 37) on the keyboard. Multiply by 46.

*Step 2* Set 54 on the keyboard and divide.

*Result* Upper dials 64, Cubic yards of fill between first two stations

Continue in the same way to calculate the other fills.

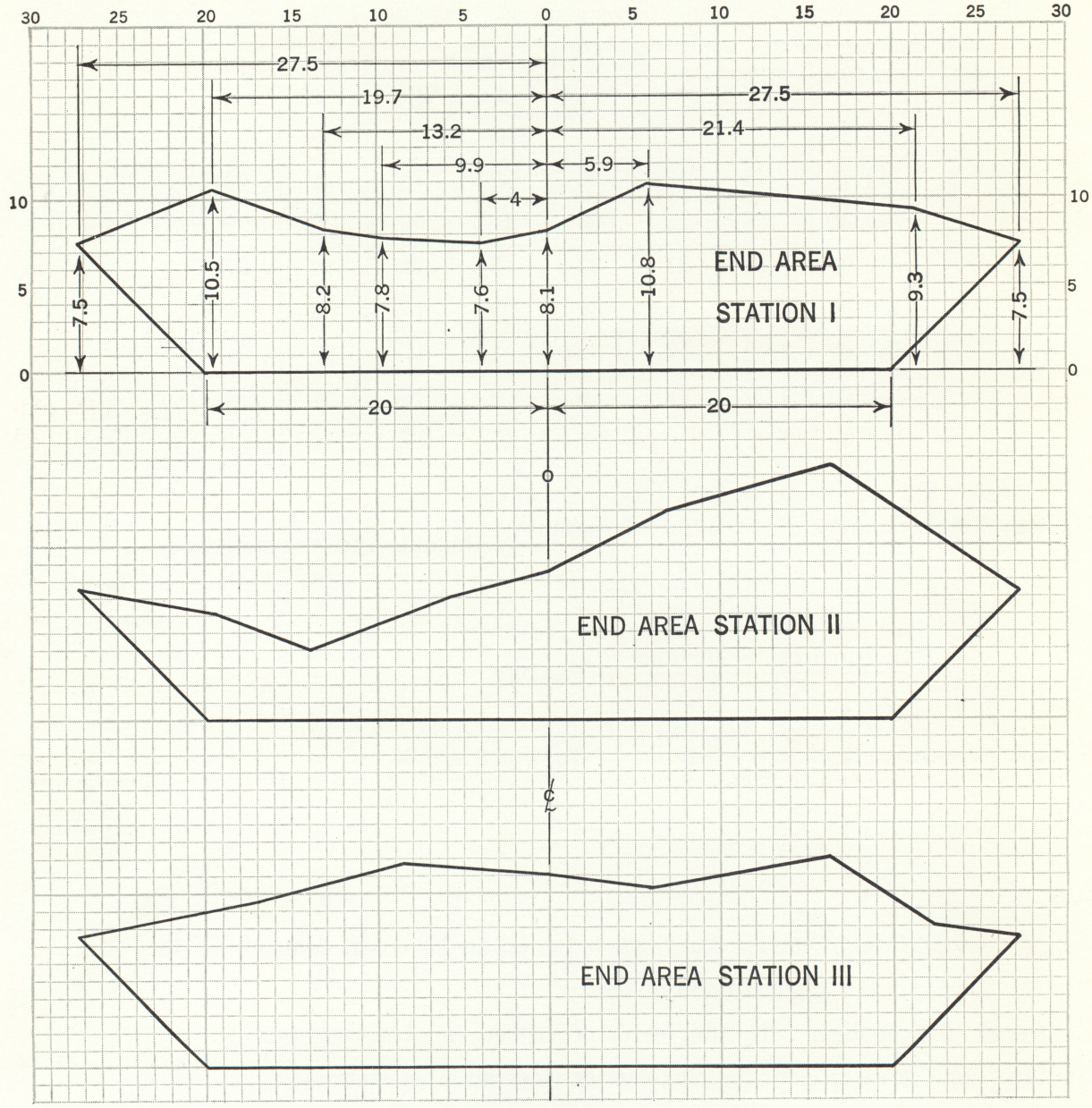
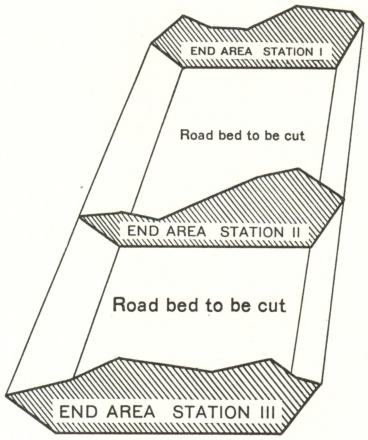

30 25 20 15 10 5 0 5 10 15 20 25 30
27.5
19.7
27.5
13.2
21.4
9.9
5.9
4
10
5
0
7.5
10.5
8.2
7.8
7.6
8.1
10.8
9.3
7.5
END AREA
STATION I
20
20
0
END AREA STATION II
℄
END AREA STATION III

**FIGURE 13**

# Cuts for Roadbed by End Area Method




The areas of the cross sections of the cut are calculated from the measurements of the altitudes from the roadbed and the distance measurements from the right and left of a center or 0 line taken at certain points on the surface. Then the cubic yardage volume of dirt to be cut in order to level the roadbed is figured.

**Formula** Volume in cubic yards $= \dfrac{(A_1 + A_2)\ L}{108}$

$A_1$ = Double end area Station I

$A_2$ = Double end area Station II

L = Distance in feet between cross sections

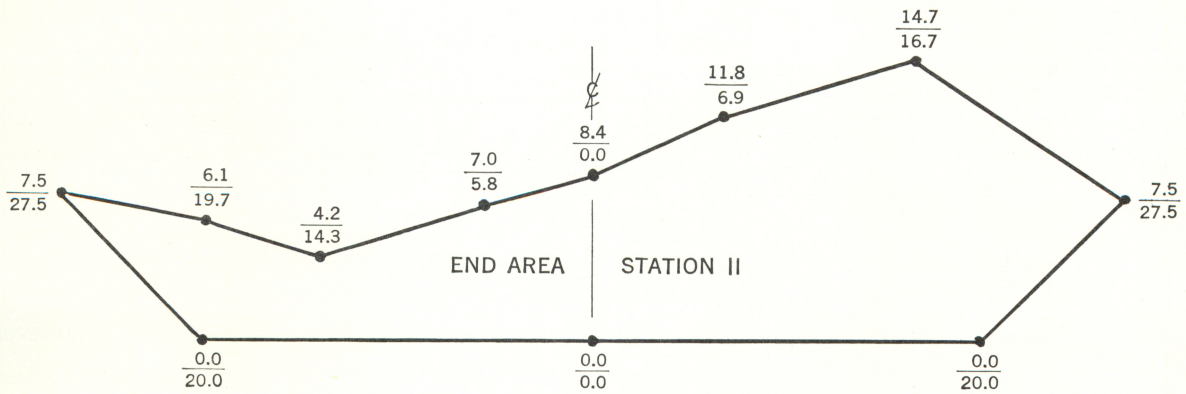
**Example** Figure 13 shows the end areas of three stations. There are 50 feet between each station. From the measurements given, calculate the end area of each and the cubic yards of dirt to be removed to form a level roadbed.

**Program** Decimals: Keyboard 2 — Tab at 3
Upper Dials 2 — Transfer Slide 2
Lower Dials 4 — AUTO KB CLEAR to Right
Non-entry On
Upper Dials Locked

The end area of Station II is found first since it is required for calculating the volumes between both the other stations.

To figure the end area the coordinates are arranged in a table starting at the upper center line point and working to the right, clockwise; then starting at the upper center line and working to the left, counterclockwise.

### Station II



| CENTER LINE | TO THE RIGHT — CLOCKWISE | | | | | CENTER LINE | TO THE LEFT — COUNTERCLOCKWISE | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 8.4 | 11.8 | 14.7 | 7.5 | 0.0 | 0.0 | 8.4 | 7.0 | 4.2 | 6.1 | 7.5 | 0.0 |
| 0.0 | 6.9 | 16.7 | 27.5 | 20.0 | 0.0 | 0.0 | 5.8 | 14.3 | 19.7 | 27.5 | 20.0 |

*Step 1* Multiply accumulatively 8.4 by 6.9. Multiply negatively 6.9 by 14.7. Multiply accumulatively 14.7 by 27.5.

*Step 2* Change keyboard set-up to 8.4 and multiply accumulatively by 5.8. Multiply negatively 5.8 by 4.2.

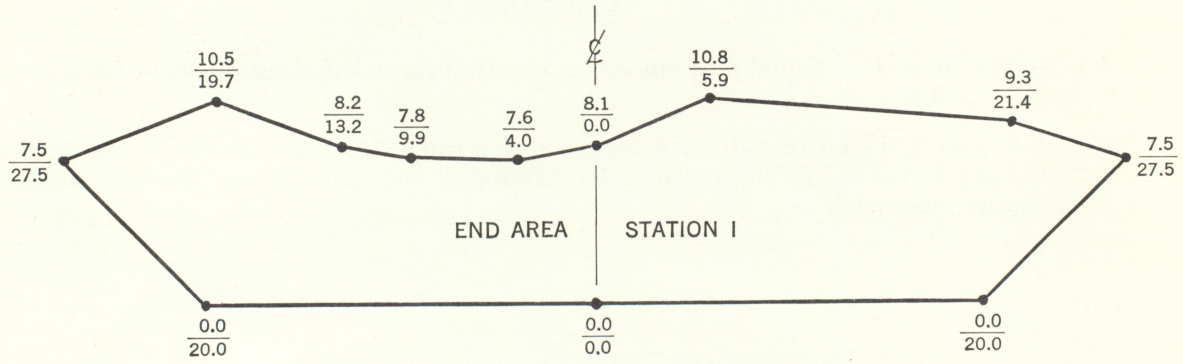
*Step 3* Multiply accumulatively 4.2 by 19.7. Multiply negatively 19.7 by 7.5. Multiply accumulatively 7.5 by 20.0.

*Step 4* Change keyboard set-up to 27.5 and multiply accumulatively by 6.1. Multiply negatively 6.1 by 14.3. Multiply accumulatively 14.3 by 7.0.

*Step 5* Change keyboard set-up to 20.0 and multiply accumulatively by 7.5. Multiply negatively 7.5 by 16.7. Multiply accumulatively 16.7 by 11.8. *Result* Lower dials 872.56, Double end area of Station II. Note this result. Do not clear machine.

## Station I

Next the double end area of Station I is calculated and added to the double end area of Station II which is 872.56, result of last step.




| CENTER LINE | TO THE RIGHT — CLOCKWISE | | | | | CENTER LINE | TO THE LEFT — COUNTERCLOCKWISE | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 8.1 | 10.8 | 9.3 | 7.5 | 0.0 | 0.0 | 8.1 | 7.6 | 7.8 | 8.2 | 10.5 | 7.5 | 0.0 |
| 0.0 | 5.9 | 21.4 | 27.5 | 20.0 | 0.0 | 0.0 | 4.0 | 9.9 | 13.2 | 19.7 | 27.5 | 20.0 |

*Step 6* Multiply accumulatively 8.1 by 5.9. Multiply negatively 5.9 by 9.3. Multiply accumulatively 9.3 by 27.5.

*Step 7* Change keyboard set-up to 8.1 and multiply accumulatively by 4.0. Multiply negatively 4.0 by 7.8.

*Step 8* Multiply accumulatively 7.8 by 13.2. Multiply negatively 13.2 by 10.5. Multiply accumulatively 10.5 by 27.5.

*Step 9* Change keyboard set-up to 20.0 and multiply accumulatively by 7.5. Multiply negatively 7.5 by 19.7. Multiply accumulatively 19.7 by 8.2.

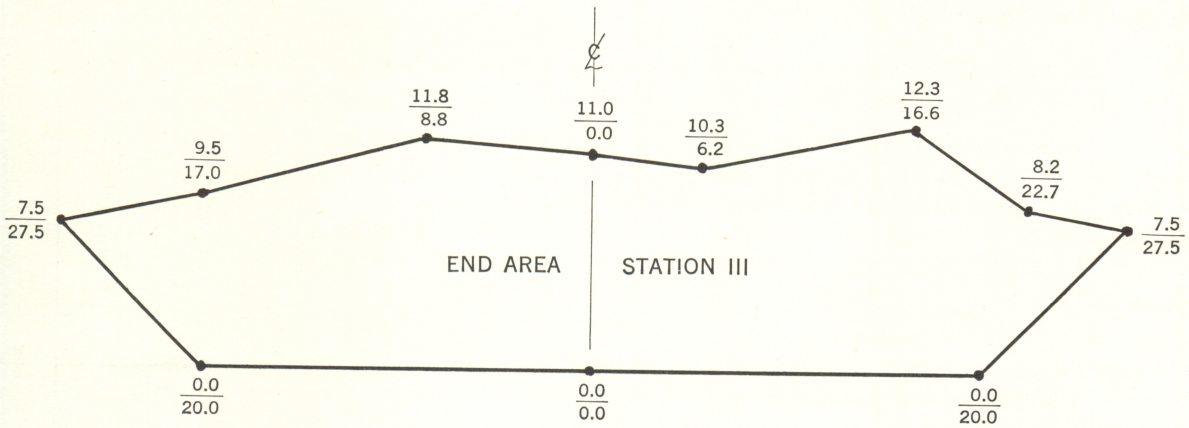
*Step 10* Multiply negatively 8.2 by 9.9. Multiply accumulatively 9.9 by 7.6.

*Step 11* Change keyboard set-up to 20.0 and multiply accumulatively by 7.5. Multiply negatively 7.5 by 21.4. Multiply accumulatively 21.4 by 10.8. *Result* Lower dials 1754.01, Total double end areas of Stations I and II.

*Step 12* Transfer and multiply by 50, distance between stations. Divide by 108.

*Result* Upper dials 812.04, Cubic yards cut between Stations I and II.

## Station III




| CENTER LINE | TO THE RIGHT — CLOCKWISE | | | | | | CENTER LINE | TO THE LEFT — COUNTERCLOCKWISE | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 11.0 | 10.3 | 12.3 | 8.2 | 7.5 | 0.0 | 0.0 | 11.0 | 11.8 | 9.5 | 7.5 | 0.0 |
| 0.0 | 6.2 | 16.6 | 22.7 | 27.5 | 20.0 | 0.0 | 0.0 | 8.8 | 17.0 | 27.5 | 20.0 |

*Step 13* Set on keyboard 872.56, double end area of Station II and depress ENTER DIVD to register in lower dials.

*Step 14* Multiply accumulatively 11.0 by 6.2. Multiply negatively 6.2 by 12.3.

*Step 15* Multiply accumulatively 12.3 by 22.7. Multiply negatively 22.7 by 7.5. Multiply accumulatively 7.5 by 20.0.

*Step 16* Change keyboard set-up to 11.8 and multiply accumulatively by 17.0. Multiply negatively 17.0 by 7.5. Multiply accumulatively 7.5 by 20.0.

*Step 17* Change keyboard set-up to 27.5 and multiply accumulatively by 9.5. Multiply negatively 9.5 by 8.8. Multiply accumulatively 8.8 by 11.0.

*Step 18* Change keyboard set-up to 27.5 and multiply accumulatively by 8.2. Multiply negatively 8.2 by 16.6.

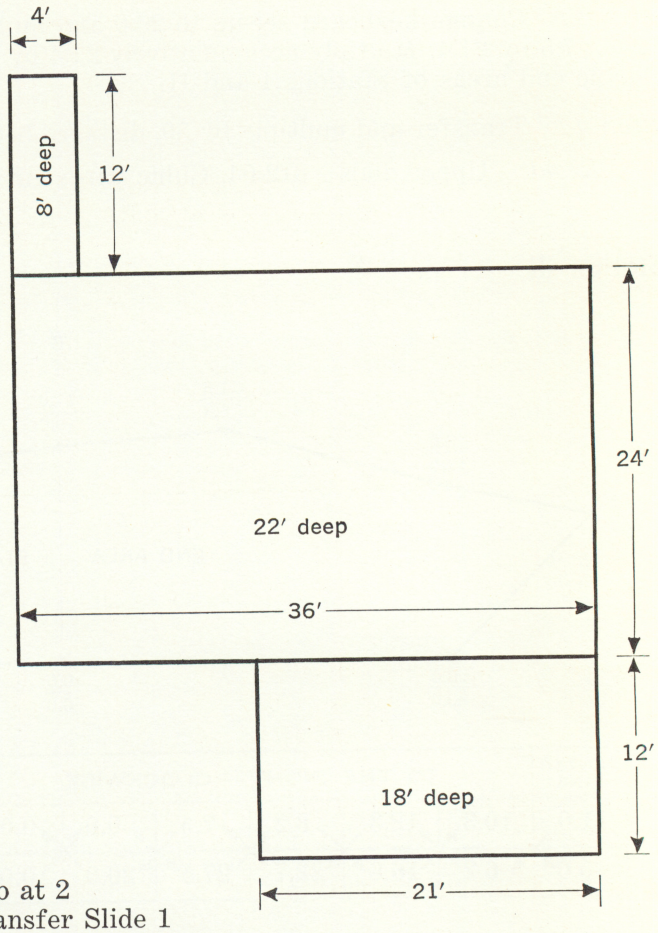
*Step 19* Multiply accumulatively 16.6 by 10.3. *Result* Lower dials 1881.37, Total double end areas of Stations II and III.

*Step 20* Transfer and multiply by 50, distance between stations. Divide by 108.

*Results* Left upper dials 871.00, Cubic yards cut between Stations II and III
Right upper dials 1683.04, Cubic yards cut between Stations I and III.

# Area and Volume

Engineers and estimators, particularly those who are in assessment departments, occasionally are required to find both square feet area and cubic feet volume. With the Monroe they can figure the two in one simultaneous operation.




| **Example** | 12 × 4 | × 8 |
|---|---|---|
| | 24 × 36 | × 22 |
| | 12 × 21 | × 18 |
| Total | 1164 Sq. | 23928 Cu. |

**Program** Decimals: Keyboard 5 Tab at 2
Upper Dials 4 Transfer Slide 1
Lower Dials 9 Upper Dials Locked
Left Lower Dials Locked

Set lengths on the extreme right of the keyboard and widths and depths at the fifth keyboard decimal.

*Step 1* Set 12, length, on the extreme right of the keyboard and multiply by 4, width. Transfer and multiply by 8, depth. *Results* Upper dials 48, square feet area; Lower dials 384, cubic feet volume.

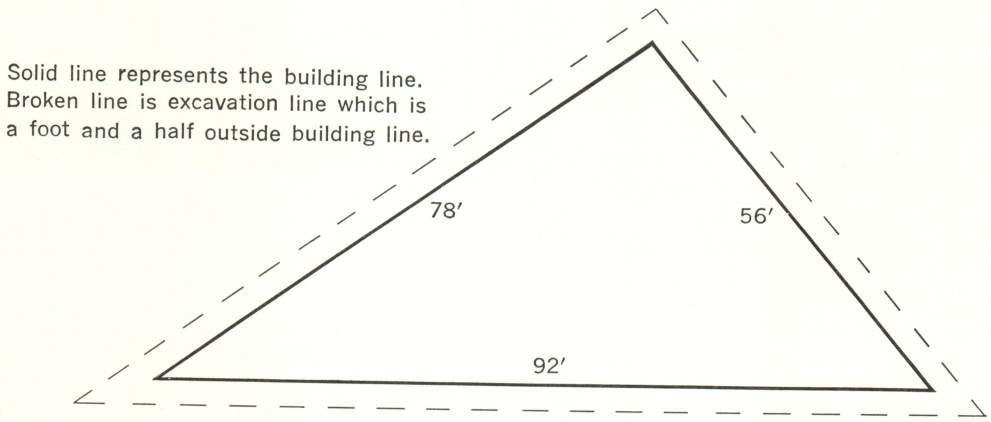
*Step 2* Set 24, length, on the extreme right of the keyboard and multiply by 36, width. Transfer and multiply by 22, depth.

*Step 3* Set 12, length, on the extreme right of the keyboard and multiply by 21, width. Transfer and multiply by 18, depth.

*Results* Upper dials 1164, Total square feet area
Lower dials 23928, Total cubic feet volume

# Excavation for Triangular Foundation

**Example** A building with the dimensions shown in the figure below is to be constructed on a triangular tract of land. If the excavation is to be nine feet deep and extend a foot and a half outside the building line, determine the volume of earth to be removed.




**Formula** $A = \sqrt{s\,(s-a)\,(s-b)\,(s-c)}$ where $s = \frac{1}{2}\,(a+b+c)$

**Program** Decimals: Keyboard 2 Tab at 3
Upper Dials 2 Transfer Slide 2
Lower Dials 4

*Step 1* Set 56, length of side a, on the keyboard and depress ENTER DIVD. Add 92 and 78, lengths of sides b and c respectively.

*Step 2* Transfer and multiply by .5. *Result* Lower dials 113, s.

*Step 3* Move repeat control to Repeat position. Set 56, a, on the keyboard and subtract from the 113, s, in the lower dials. *Result* Lower dials 57, s — a. Depress plus bar. Set 92, b, on the keyboard and subtract. *Result* Lower dials 21, s — b. Depress plus bar. Set 78, c, on the keyboard and subtract. *Result* Lower dials 35, s — c.

*Step 4* Move repeat control to Non-repeat position. Multiply 113, s, by 57, s — a. Transfer and multiply by 21, s — b. Transfer and multiply by 35, s — c. *Result* Lower dials 4734135.

*Step 5* Find the square root of 4734135 by the Monroe Simplified Method.

*Result* 2175.80, Sq. ft., Area of inner triangle

Determining the dimensions of the larger triangle represented by the excavation line is unnecessary. To find the area of the foot and a half strip around the triangular building line, arbitrarily add 4.5 feet to the total of the sides of the foundation wall.

*Step 6* Add 56, 92, 78, and 4.5. Transfer and multiply by 1.5.

*Result* Lower dials 345.75, Sq. ft., Area of outer triangle

*Step 7* Add 345.75, area of outer triangle, and 2175.80, area of inner triangle. Transfer and multiply by 9, depth. Divide by 27, cubic feet in a cubic yard.

*Result* Upper dials 840.51, Cu. yds., Total volume to be excavated

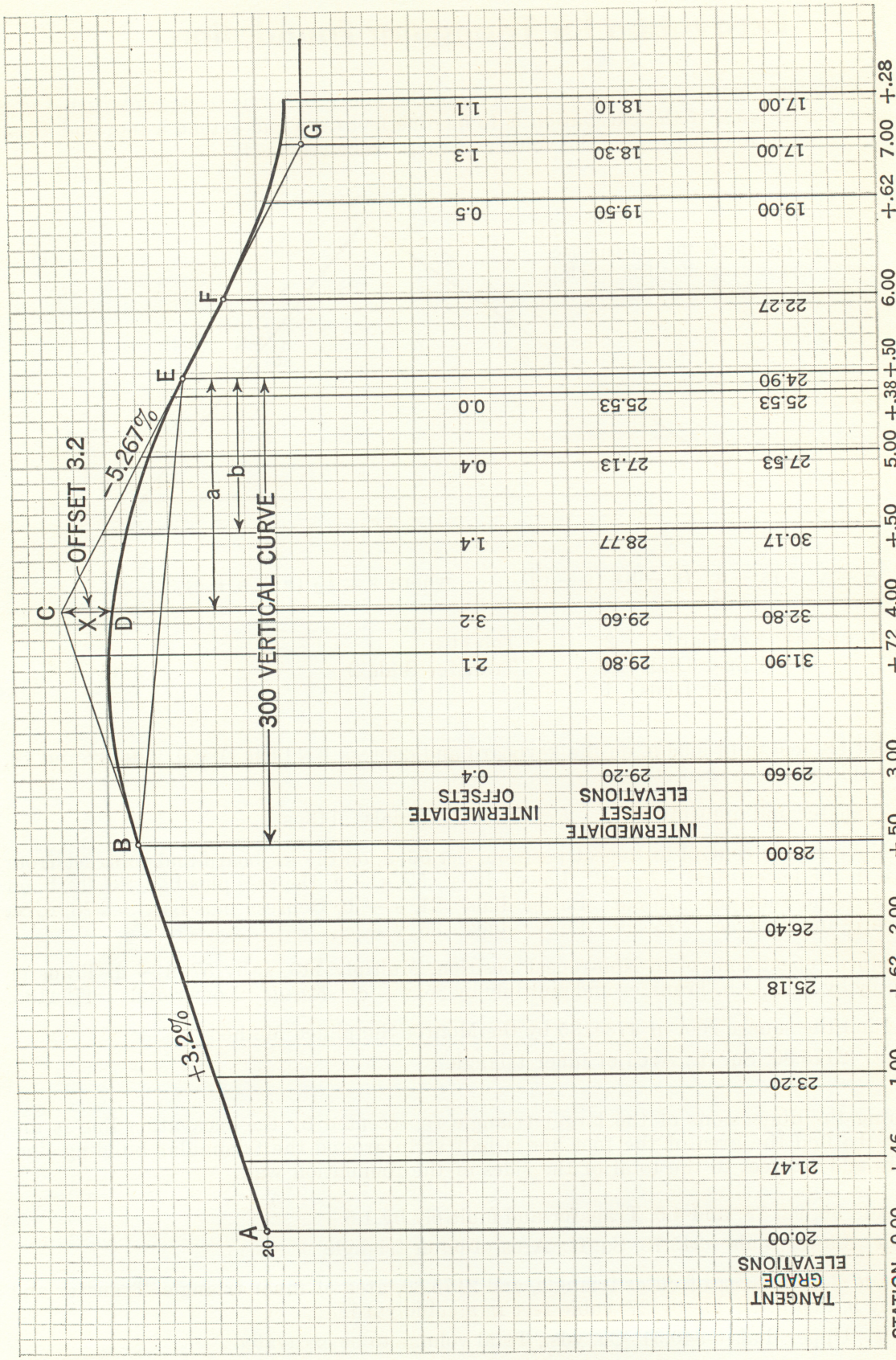

STATION
TANGENT GRADE ELEVATIONS
INTERMEDIATE OFFSET ELEVATIONS
INTERMEDIATE OFFSETS
300 VERTICAL CURVE
OFFSET 3.2
−5.26⅞%
+3.2%
A
B
C
D
E
F
G
X
a
b
20
0.00
+.46
1.00
+.62
2.00
+.50
3.00
+.72
4.00
+.50
5.00
+.38
+.50
6.00
+.62
7.00
+.28
20.00
21.47
23.20
25.18
26.40
28.00
29.60
31.90
32.80
30.17
27.53
25.53
24.90
22.27
19.00
17.00
17.00
17.00
29.20
29.80
29.60
28.77
27.13
25.53
19.50
18.30
18.10
0.4
2.1
3.2
1.4
0.4
0.0
0.5
1.3
1.1

FIGURE 15

# Vertical Curve

**Example** Find the grades, elevations, and offset for the vertical curve in Figure 15.

## TANGENT GRADES

Tangent grade lines are straight lines roughly conforming to the present slopes of the ground involved. The point of intersection of these tangent grades is termed the P.I.

**Formula** $$\frac{\text{Difference in elevation at P.I.}}{\text{Distance between stations}} = \%\ \text{Grade}$$

**Program** Decimals: Keyboard 5 Tab at 6
Upper Dials 5
Lower Dials 10

**Grade A-C** $$\frac{\text{Elevation at C} - \text{Elevation at A}}{\text{Distance A to C}}$$

*Step 1* Set 32.8, elevation at C, on the keyboard and depress ENTER DIVD to register in lower dials correctly pointed off. Set 20, elevation at A, on the keyboard and subtract.

*Step 2* Set 4.0, distance A-C, on the keyboard and divide.

*Result* Upper dials 3.200 or 3.2% Grade A-C

**Grade C-G** $$\frac{\text{Elevation at C} - \text{Elevation at G}}{\text{Distance C to G}}$$

*Step 3* Set 32.8 on the keyboard and depress ENTER DIVD. Set 17.0 on the keyboard and subtract.

*Step 4* Set 3.0 on the keyboard and divide.

*Result* Upper dials 5.26666 or − 5.267%, Grade C-G

The value is given the minus sign as the slope is downward from C to G.

## TANGENT GRADE ELEVATIONS

The grades having been determined, the tangent grade elevation for each station is calculated, using the difference in distance between it and the next station.

| POSITIVE GRADE | | NEGATIVE GRADE | |
|---|---|---|---|
| STATIONS | DIFFERENCE | STATIONS | DIFFERENCE |
| 0.00 — .46 | .46 | 4.00 — 4.50 | — .50 |
| .46 — 1.00 | .54 | 4.50 — 5.00 | — .50 |
| 1.00 — 1.62 | .62 | 5.00 — 5.38 | — .38 |
| 1.62 — 2.00 | .38 | 5.38 — 5.50 | — .12 |
| 2.00 — 2.50 | .50 | 5.50 — 6.00 | — .50 |
| 2.50 — 3.00 | .50 | 6.00 — 6.62 | — .62 |
| 3.00 — 3.72 | .72 | 6.62 — 7.00 | — .38 |
| 3.72 — 4.00 | .28 | 7.00 — 7.28 | — .28 |

With the upward grade all calculations are positive; with the downward grade, calculations are negative.

**Stations A to C**

*Step 1* Set 20, beginning elevation at A, on the keyboard and depress ENTER DIVD to register in the lower dials. Set 3.2, tangent grade A-C, on the keyboard, enter and lock as constant multiplier by depressing ENTER MULTIPLIER, then moving down the constant multiplier lever.

*Step 2* Multiply accumulatively by .46, difference between first and second stations.

*Result* Lower dials 21.47, Elevation at second station

*Step 3* Multiply accumulatively by .54, difference between second and third stations.

*Result* Lower dials 23.20, Elevation at third station

*Step 4* Multiply accumulatively by .62, difference between third and fourth stations.

*Result* Lower dials 25.18, Elevation at fourth station

Continue in the same way to calculate the tangent grade elevations for the rest of the stations, fifth to ninth. Before starting the multiplication operation for the ninth station, raise the constant multiplier lever. At this point the amount in the lower dials is 32.8 which is the elevation at C.

**Stations C to G**

*Step 5* Set 5.267, tangent grade C-G on the keyboard, enter and lock as constant multiplier by moving down constant multiplier lever.

*Step 6* Multiply negatively by .50, difference between ninth and tenth stations.

*Result* Lower dials 30.17, Elevation at tenth station

*Step 7* Multiply negatively by .50, difference between tenth and eleventh stations.

*Result* Lower dials 27.53, Elevation at eleventh station

*Step 8* Multiply negatively by .38, difference between eleventh and twelfth stations.

*Result* Lower dials 25.53, Elevation at twelfth station

Continue multiplying negatively in the same way for the rest of the stations. Before starting the last multiplication for the final station, raise the constant multiplier lever.

## OFFSETS

At the intersections of tangent grades there are inserted vertical curves (VC's) which generally extend equal distances on each side of the intersection. To locate these vertical curves it is necessary first to find the offset or external opposite the P.I. (point D in the diagram, Figure 15) and the intermediate offsets. The offset is halfway between the line B-E and point C (X in Figure 15).

**Formula** $$\frac{\text{Algebraic difference in grades} \times \text{Length of curve in stations}}{\text{Number of stations}} = \text{Offset}$$

*Step 1* Set transfer slide at 5. Set 3.20, grade A-C, on keyboard and depress ENTER DIVD to register in lower dials. Set 5.267 on keyboard and add.

*Step 2* Transfer and multiply by 3.0, the distance between stations 2.50 and 5.50.

*Step 3* Divide by 8, number of stations.

*Result* Upper dials 3.17512 or 3.2, Offset

### Intermediate Offsets

**Formula** $\frac{b}{\frac{a^2}{X}} \times b =$ Intermediate offset

a = ½ length of the vertical curve

b = Distance from station on curve to offset, or distance from offset to station

X = Offset

*Step 1* Set ½ the length of the vertical curve on the keyboard as 1.50 and square. Divide by 3.2, offset. *Result* Upper dials .70312, $\frac{a^2}{X}$

*Step 2* Square .50, distance between first intermediate station 2.50 and station 3.00. Divide by .70312.

*Result* Upper dials .4, Intermediate offset for station 3.00

*Step 3* Square 1.22, distance between stations 2.50 and 3.72. Divide by .70312.

*Result* Upper dials 2.1, Intermediate offset for station 3.72

*Step 4* Square 1.00, distance between stations 4.50 and 5.50. Divide by .70312.

*Result* Upper dials 1.4, Intermediate offset for station 4.50

*Step 5* Square .50, distance between stations 5.00 and 5.50. Divide by .70312.

*Result* Upper dials .4, Intermediate offset for station 5.00

*Step 6* Square .12, distance between stations 5.38 and 5.50. Divide by .70312.

*Result* Upper dials .020, Intermediate offset is so slight it can be disregarded

### Intermediate Offset Elevations

Subtracting the intermediate offset from the tangent grade elevation determines the intermediate offset elevation.

| STATION | TANGENT GRADE ELEVATION | INTERMEDIATE OFFSET ELEVATION | INTERMEDIATE OFFSET |
|---|---|---|---|
| 2.50 | 28.00 | 28.00 | |
| 3.00 | 29.60 | 29.20 | 0.4 |
| 3.72 | 31.90 | 29.80 | 2.1 |
| 4.00 | 32.80 | 29.60 | 3.2 (offset) |
| 4.50 | 30.17 | 28.77 | 1.4 |
| 5.00 | 27.53 | 27.13 | 0.4 |
| 5.38 | 25.53 | 25.53 | 0.0 (slight) |
| 5.50 | 24.90 | 24.90 | |

# Length of Arc by Radians

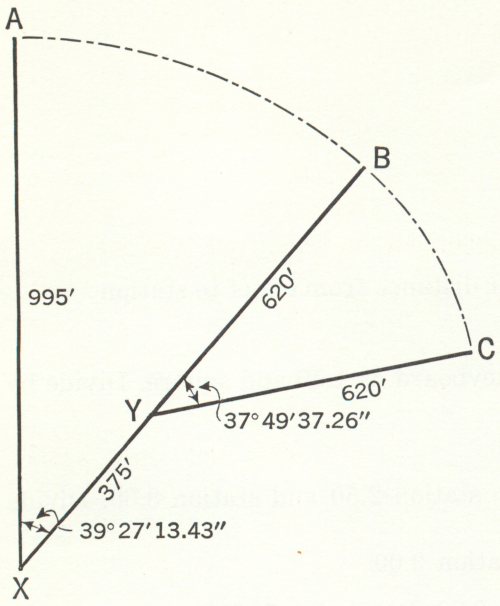



FIGURE 16

A curve length can be determined by means of the formula for measurement of the circumference of a circle. However, a method by radian measure is now almost universally used for finding arc length. The radian measure of an angle is the ratio of the arc it subtends to the radius of the circle in which it is the central angle.

**Example** Find the lengths of arc A-B and arc B-C in Figure 16 by the arithmetic method and the table method.

**Program**

| | | | |
|---|---|---|---|
| Decimals: | Keyboard | 3 | Tab at 4 |
| | Upper Dials | 3 | Transfer Slide 3 |
| | Lower Dials | 6-13 | |

## Arithmetic Method

The degrees and minutes are converted to seconds and multiplied first by the radian measure of 1 second and then by the radius of the circle.

### Length of Arc A-B

*Step 1* Multiply 39, degrees, by 3600, the seconds in 1 degree.

*Step 2* Multiply accumulatively 27, minutes, by 60, the seconds in 1 minute.

*Step 3* Add 13.43, seconds. *Result* Lower dials 142033.43, Total seconds for angle A-X-B.

*Step 4* Transfer and multiply by .0000048481, length of arc for 1 second of circle radius 1. set on the extreme right of the keyboard.

*Step 5* Transfer and multiply by 995, radius.

*Result* Lower dials at 13th decimal 685.15, Length of Arc A-B

### Length of Arc B-C

*Step 6* Multiply 37 by 3600.

*Step 7* Multiply accumulatively 49 by 60.

*Step 8* Add 37.26, seconds. *Result* Lower dials 136177.26, Total seconds for angle B-Y-C.

*Step 9* Transfer and multiply by .0000048481 set on extreme right of keyboard.

*Step 10* Transfer and multiply by 620.

*Result* Lower dials at 13th decimal 409.32, Length of arc B-C

## Table Method

In this method the increments of the radian measure of the angle are added and multiplied by the radius of the circle. The radian values for degrees, minutes, seconds, and hundredths of seconds are taken from the Monroe Table for Radian Measure (Form 1131-S).

The program and decimal set-up are the same as above.

### Length of Arc A-B

*Step 1* Set .6806784083, value from table for 39°, on the extreme right of the keyboard and depress ENTER DIVD.

*Step 2* Add .0078539816, value from table for 27′. Add .0000630258, value from table for 13″. Add .0000020847, value from table for .43″.

*Step 3* Transfer and multiply by 995, radius.

*Result* Lower dials at 13th decimal 685.15, Length of arc A-B

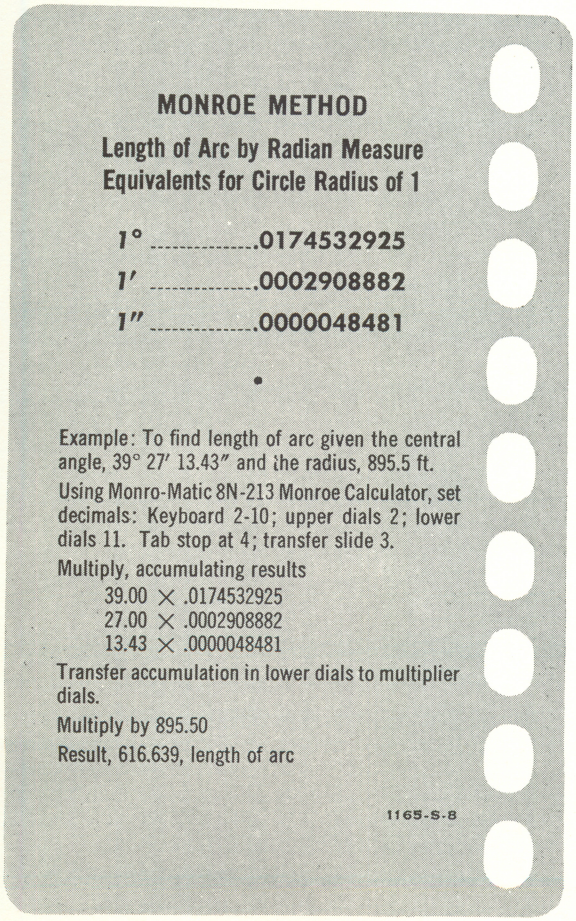
### Length of Arc B-C

*Step 4* Set .6457718232, value for 37°, on the extreme right of the keyboard. Depress ENTER DIVD.

*Step 5* Add .0142535222, value for 49′. Add .0001793811, value for 37″. Add .0000012605, value for .26″.

*Step 6* Transfer and multiply by 620, radius.

*Result* Lower dials at 13th decimal 409.33, Length of arc B-C




## Method No. 3

A third method is based upon the predetermined arc length by radian measure equivalents for a circle radius of 1. These equivalent factors are given on the Monroe Keyboard Card (1165-S-8).

**Example** Find the length of arc with the central angle 51° 39′ 14.74″ and radius 1152 feet.

### Program

| | | | |
|---|---|---|---|
| Decimals: | Keyboard | 2 | Tab at 4 |
| | Upper Dials | 2 | Transfer Slide 3 |
| | Lower Dials | 11 | |

*Step 1* Multiply 51, degrees, by .0174532925, equivalent for 1° from Monroe Keyboard Card.

*Step 2* Multiply accumulatively 39, minutes, by .0002908882, equivalent for 1′.

*Step 3* Multiply accumulatively 14.74, seconds, by .0000048481, equivalent for 1″.

*Step 4* Transfer and multiply by 1152, radius.

*Result* Lower dials 1038.567, Arc length

*The handy plastic card (Monroe Form 1165-S-8), reproduced here, fits on the left-hand side of the keyboard of the Monroe Calculator. It will be furnished free by any local office of the company or its general offices.*

## Length of Arc by Radians — Continued

### Method No. 4

**Formula** $L = \frac{(3.14159)\ R\ \Theta^\circ}{180}$

**Example** The field layout of a roadbed shows that the radius is 995 feet and the angle is 39° 27′ 13.43″. Find the arc length.

**Program**

| | | | |
|---|---|---|---|
| Decimals: | Keyboard | 5 | Tab at 6 |
| | Upper Dials | 5 | Transfer Slide 5 |
| | Lower Dials | 10 | |

*Step 1* Multiply 13.43, seconds, by .01667, equivalent for 1/60th.

*Step 2* Add 27, minutes. Transfer and multiply by .01667, equivalent for 1/60th.

*Step 3* Add 39, degrees. *Result* Lower dials 39.45382, Decimal equivalent for 39° 27′ 13.43″.

*Step 4* Transfer and multiply by 3.14159.

*Step 5* Transfer and multiply by 995, radius.

*Step 6* Divide by 180.

*Result* Upper dials 685.16, Arc length

# Highway Curve

## CURVATURE FOR SPEED OF TRAVEL

The degree of curvature in a highway curve is according to the intended speed of travel over the road. Safety through super-elevation or banking determines the radius of the curve.

**Formula** $R = 0.258\ V^2$

R = Radius of circular curve
V = Velocity in mph
0.258 = Constant factor which takes into account pull of gravity, tire friction, maximum super-elevation slope of 0.10.

**Example** Find the radius of a curve for safe driving at 50 mph.

**Program** Decimals: Keyboard 3 Tab at 4
Upper Dials 3 Transfer Slide 3
Lower Dials 6

*Step 1* Set 50 on the keyboard and square.

*Step 2* Transfer and multiply by 0.258.

*Result* Lower dials 645, Radius of circular curve

## SPIRAL CURVE

The design engineer uses a spiral curve to develop gradually the degree of curvature in a circular curve. It is inserted between the circular curve and the tangent and is of varying radii. The end adjacent to the tangent has the longer radius and the radius decreases gradually as the spiral curve approaches the circular curve. Where it joins the circular curve the radii of the two curves are equal. A spiral curve is sometimes called an easement curve or transition curve.

**Formula** $L_S = \dfrac{1.6\ V^3}{R}$

$L_S$ = Length of spiral curve
V = Velocity in mph
R = Radius of circular curve

**Example** Find the length of the spiral curve when the velocity is 50 and the radius is 645.

*Step 1* Cube 50 by setting on the keyboard, squaring, then transfer multiply to cube.

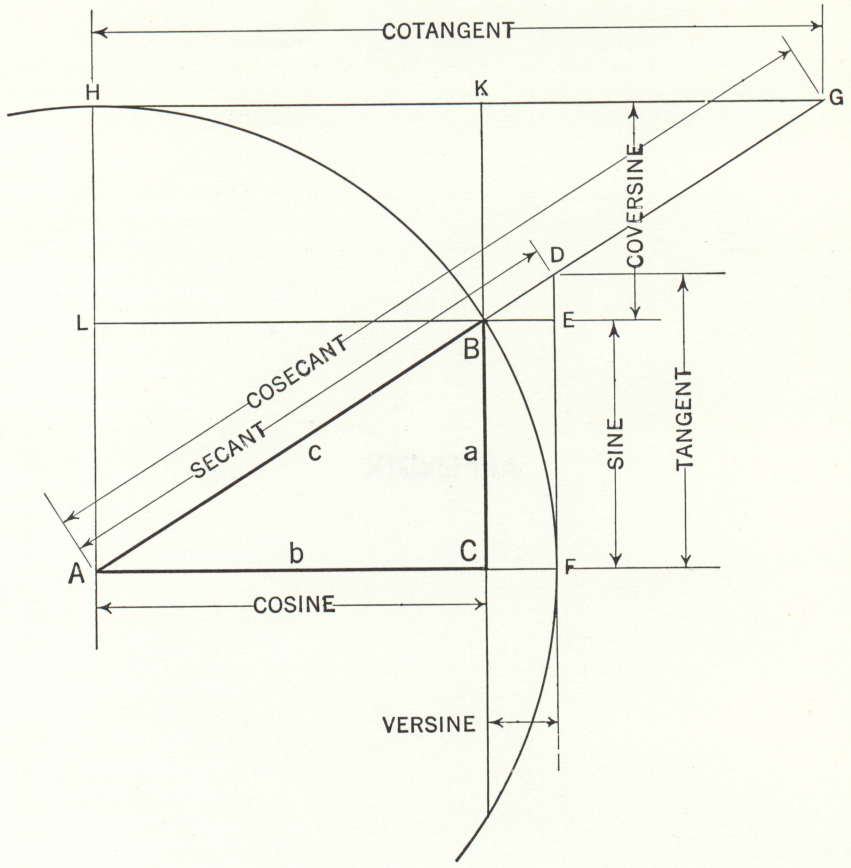
*Step 2* Transfer and multiply by 1.6.

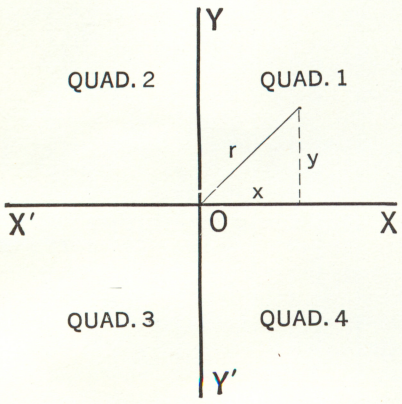
*Step 3* Divide by 645.

*Result* Upper dials 310.077, Length of spiral curve

# APPENDIX

## THE RIGHT ANGLE TRIANGLE




| QUAD | SIN | COS | TAN | COT | SEC | CSEC |
|---|---|---|---|---|---|---|
| NO. 1 | + | + | + | + | + | + |
| NO. 2 | + | — | — | — | — | + |
| NO. 3 | — | — | + | + | — | — |
| NO. 4 | — | + | — | — | + | — |

## THE RIGHT ANGLE TRIANGLE

$c^2 = a^2 + b^2$

$c = \sqrt{a^2 + b^2}$

$b^2 = c^2 - a^2$

$b = \sqrt{(c + a)\ (c - a)}$

$\text{Sin A} = \frac{a}{c} = \text{Cos B}$

$\text{Cos A} = \frac{b}{c} = \text{Sin B}$

$\text{Tan A} = \frac{a}{b} = \text{Cot B}$

$\text{Cosec A} = \frac{c}{a} = \text{Sec B}$

$\text{Sec A} = \frac{c}{b} = \text{Cosec B}$

$\text{Cot A} = \frac{b}{a} = \text{Tan B}$

$\text{Vers A} = \frac{c - b}{c} = \text{Covers B}$

$\text{Exsec A} = \frac{c - b}{b} = \text{Coexsec B}$

$A + B + C = 180°$

$a^2 = c^2 - b^2$

$a = \sqrt{(c + b)\ (c - b)}$

$A + B = C = 90°$

$A = 90° - B$

$\text{Covers A} = \frac{c - a}{c} = \text{Versin B}$

$\text{Coexsec A} = \frac{c - a}{a} = \text{Exsec B}$

$a = c \text{ Sin A} = b \text{ Tan A}$

$a = c \text{ Cos B} = b \text{ Cot B}$

$b = c \text{ Cos A} = a \text{ Cot A}$

$b = c \text{ Sin B} = a \text{ Tan B}$

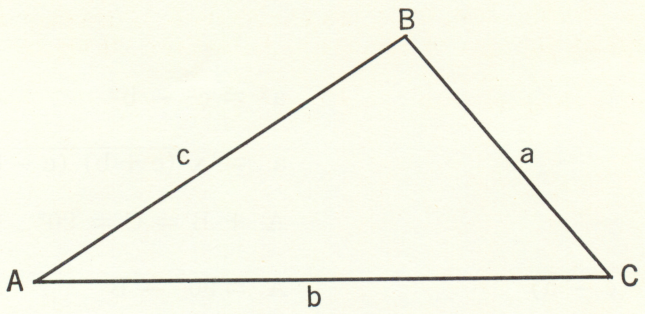
$c = \frac{a}{\text{Sin A}} = \frac{b}{\text{Cos A}}$

$c = \frac{a}{\text{Cos B}} = \frac{b}{\text{Sin B}}$

$\frac{a}{\text{Sin A}} = \frac{b}{\text{Sin B}} = \frac{c}{\text{Sin C}}$

$$\text{Area} = \frac{ab}{2}$$

## THE OBLIQUE TRIANGLE




**Case 1** Given one side a and two angles A, B, the third angle C is found from $A + B + C = 180°$ and the other two sides b and c by the law of sines.

$$C = 180° - (A + B) \qquad b = \frac{a}{\text{Sin } A} \times \text{Sin } B$$

$$c = \frac{a}{\text{Sin } A} \times \text{Sin } (A + B) = \frac{a}{\text{Sin } A} \times \text{Sin } C$$

$$\text{Area} = \tfrac{1}{2}\, ab \text{ Sin } C = \frac{a^2 \text{ Sin } B \text{ Sin } C}{2 \text{ Sin } A}$$

**Case 2** Given two sides a, b, and the angle opposite one of them A, the angle opposite the other given side B is found by the law of sines; the third angle C is found from the relation $A + B + C = 180°$; the third side c is found by the law of sines.

$$\text{Sin } B = \frac{\text{Sin } A}{a} \times b \qquad C = 180° - (A + B)$$

$$c = \frac{a}{\text{Sin } A} \times \text{Sin } C = \frac{b}{\text{Sin } B} \times \text{Sin } C$$

$$\text{Area} = \tfrac{1}{2}\, ab \text{ Sin } C$$

**Case 3** Given two sides a, b, and the included angle C the third side c is found by the law of cosines and the remaining angles B, A by either the law of cosines or the law of sines.

$$c = \sqrt{a^2 + b^2 - 2\, ab \text{ Cos } C}$$

$$\text{Sin } A = \frac{a}{c} \times \text{Sin } C \qquad B = 180° - (A + C)$$

$$\text{Cos } B = \frac{a^2 + c^2 - b^2}{2\, ac}$$

$$\text{Cos } A = \frac{b^2 + c^2 - a^2}{2\, bc}$$

$$\text{Area} = \tfrac{1}{2}\, ab \text{ Sin } C$$

## THE OBLIQUE TRIANGLE

**Case 4** Given the three sides a, b, c, the three angles A, B, C are found by the law of cosines; or one angle A is found by the law of cosines and then the others by the law of sines.

In the following $s = \frac{1}{2}(a + b + c)$

$$\text{Sin } \tfrac{1}{2} A = \sqrt{\frac{(s-b)(s-c)}{bc}}$$

$$\text{Tan } \tfrac{1}{2} B = \sqrt{\frac{(s-a)(s-c)}{s(s-b)}}$$

$$\text{Cos } A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\text{Cos } B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$C = 180° - (A + B)$$

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

# MONROE TABLE OF FACTORS FOR EXTRACTING SQUARE ROOT

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point off answer

Same decimal pointing as per column A

| N′ + | A ÷ | D |
|---|---|---|
| 1.000 | 102 | 202 |
| 1.045 | 1067 | 2066 |
| 1.09 | 1114 | 2111 |
| 1.14 | 1162 | 2156 |
| 1.19 | 1211 | 2201 |
| 1.24 | 126 | 2245 |
| 1.28 | 1303 | 2283 |
| 1.327 | 1356 | 2329 |
| 1.385 | 1416 | 238 |
| 1.45 | 1481 | 2434 |
| 1.515 | 1545 | 2486 |
| 1.575 | 1609 | 2537 |
| 1.645 | 1677 | 259 |
| 1.705 | 1741 | 2639 |
| 1.777 | 1817 | 2696 |
| 1.855 | 1896 | 2754 |
| 1.94 | 1974 | 281 |
| 2.01 | 2042 | 2858 |
| 2.08 | 2114 | 2908 |
| 2.16 | 2177 | 2951 |
| 2.222 | 2259 | 3006 |
| 2.29 | 2327 | 3051 |
| 2.38 | 2421 | 3112 |
| 2.47 | 252 | 3175 |
| 2.57 | 2621 | 3238 |
| 2.676 | 2729 | 3304 |
| 2.78 | 2829 | 3364 |
| 2.885 | 2936 | 3427 |
| 2.99 | 3038 | 3486 |
| 3.09 | 3154 | 3552 |
| 3.22 | 3285 | 3625 |
| 3.352 | 3404 | 369 |
| 3.457 | 3523 | 3754 |
| 3.59 | 3648 | 382 |
| 3.71 | 3777 | 3887 |
| 3.80 | 3863 | 3931 |
| 3.94 | 3996 | 3998 |
| 4.05 | 4056 | 4028 |
| 4.13 | 4188 | 4093 |
| 4.26 | 432 | 4157 |
| 4.40 | 4471 | 4229 |
| 4.54 | 4618 | 4298 |
| 4.705 | 4785 | 4375 |
| 4.87 | 4946 | 4448 |
| 5.035 | 513 | 453 |
| 5.225 | 5306 | 4607 |
| 5.39 | 5459 | 4673 |
| 5.56 | 565 | 4754 |
| 5.74 | 5832 | 483 |
| 5.93 | 6017 | 4906 |
| 6.12 | 62 | 498 |
| 6.29 | 6383 | 5053 |

| N′ | A | D |
|---|---|---|
| 6.29 | 6383 | 5053 |
| 6.49 | 6579 | 513 |
| 6.69 | 6791 | 5212 |
| 6.905 | 7017 | 5298 |
| 7.132 | 7244 | 5383 |
| 7.35 | 745 | 5459 |
| 7.56 | 7681 | 5543 |
| 7.80 | 7924 | 563 |
| 8.05 | 8168 | 5716 |
| 8.285 | 8404 | 5798 |
| 8.53 | 8661 | 5886 |
| 8.795 | 8922 | 5974 |
| 9.00 | 9102 | 6034 |
| 9.24 | 9345 | 6114 |
| 9.485 | 9619 | 6203 |
| 9.77 | 9894 | 6291 |
| 10.0 | 1009 | 6353 |
| 10.2 | 1032 | 6425 |
| 10.45 | 1053 | 649 |
| 10.6 | 1066 | 653 |
| 10.8 | 1086 | 6591 |
| 11.0 | 11122 | 667 |
| 11.2 | 1136 | 6741 |
| 11.5 | 1157 | 6803 |
| 11.7 | 1183 | 6879 |
| 11.9 | 1202 | 6934 |
| 12.1 | 1218 | 698 |
| 12.3 | 1245 | 7057 |
| 12.5 | 1262 | 7105 |
| 12.75 | 1287 | 7175 |
| 12.9 | 1305 | 7225 |
| 13.2 | 13286 | 729 |
| 13.4 | 1352 | 7354 |
| 13.7 | 1379 | 7427 |
| 13.9 | 1401 | 7486 |
| 14.1 | 1419 | 7534 |
| 14.3 | 1436 | 7579 |
| 14.5 | 146 | 7642 |
| 14.7 | 1488 | 7715 |
| 15.08 | 1528 | 7818 |
| 15.45 | 1563 | 7907 |
| 15.8 | 1596 | 799 |
| 16.0 | 1616 | 804 |
| 16.35 | 1652 | 8129 |
| 16.7 | 169 | 8222 |
| 17.1 | 1721 | 8297 |
| 17.35 | 1756 | 8381 |
| 17.75 | 1788 | 8457 |
| 18.05 | 1819 | 853 |
| 18.4 | 1858 | 8621 |
| 18.8 | 1897 | 8711 |
| 19.2 | 1932 | 8791 |

| N′ | A | D |
|---|---|---|
| 19.2 | 1932 | 8791 |
| 19.55 | 1978 | 8895 |
| 20.0 | 2016 | 898 |
| 20.3 | 2043 | 904 |
| 20.65 | 2088 | 9139 |
| 21.0 | 211 | 9187 |
| 21.3 | 2152 | 9278 |
| 21.7 | 2186 | 9351 |
| 22.1 | 2233 | 9451 |
| 22.5 | 227 | 9529 |
| 22.95 | 2316 | 9625 |
| 23.3 | 2343 | 9681 |
| 23.6 | 2379 | 9755 |
| 24.0 | 2427 | 9853 |
| 24.55 | 248 | 996 |
| 25.0 | 252 | 1004 |
| 25.5 | 2574 | 10147 |
| 26.0 | 2626 | 10249 |
| 26.55 | 26832 | 1036 |
| 27.1 | 2741 | 10471 |
| 27.72 | 27984 | 1058 |
| 28.25 | 2851 | 10679 |
| 28.8 | 29052 | 1078 |
| 29.3 | 2955 | 10872 |
| 29.8 | 3003 | 1096 |
| 30.2 | 3047 | 1104 |
| 30.8 | 31136 | 1116 |
| 31.45 | 31753 | 1127 |
| 32.05 | 3237 | 11379 |
| 32.7 | 33062 | 115 |
| 33.44 | 33698 | 1161 |
| 33.9 | 34222 | 117 |
| 34.6 | 3491 | 11817 |
| 35.2 | 35402 | 119 |
| 35.78 | 3606 | 1201 |
| 36.3 | 36602 | 121 |
| 37.0 | 37271 | 1221 |
| 37.5 | 37822 | 123 |
| 38.2 | 38502 | 1241 |
| 38.8 | 39062 | 125 |
| 39.4 | 3974 | 12608 |
| 40.0 | 40322 | 127 |
| 40.7 | 4103 | 12811 |
| 41.45 | 41796 | 1293 |
| 42.1 | 4251 | 1304 |
| 42.9 | 4323 | 1315 |
| 43.7 | 4389 | 1325 |
| 44.3 | 44689 | 1337 |
| 45.0 | 4536 | 1347 |
| 45.7 | 4609 | 13578 |
| 46.5 | 46922 | 137 |
| 47.35 | 47679 | 1381 |

| N′ | A | D |
|---|---|---|
| 47.35 | 47679 | 1381 |
| 48.0 | 48302 | 139 |
| 48.75 | 4914 | 1402 |
| 49.5 | 4985 | 14121 |
| 50.35 | 5073 | 14245 |
| 51.1 | 5148 | 1435 |
| 51.8 | 522 | 1445 |
| 52.7 | 5318 | 14585 |
| 53.6 | 54022 | 147 |
| 54.5 | 54908 | 1482 |
| 55.2 | 55502 | 149 |
| 56.0 | 564 | 1502 |
| 56.83 | 5738 | 1515 |
| 57.9 | 5843 | 15288 |
| 59.0 | 59444 | 1542 |
| 59.85 | 6038 | 15541 |
| 60.95 | 61465 | 1568 |
| 62.0 | 6252 | 15814 |
| 63.1 | 636 | 1595 |
| 64.0 | 644 | 1605 |
| 65.0 | 6548 | 16184 |
| 66.0 | 66585 | 1632 |
| 67.2 | 6765 | 1645 |
| 68.25 | 6879 | 16588 |
| 69.3 | 69889 | 1672 |
| 70.4 | 7098 | 1685 |
| 71.4 | 7191 | 1696 |
| 72.5 | 73102 | 171 |
| 73.73 | 7439 | 1725 |
| 75.0 | 7549 | 17377 |
| 76.0 | 76562 | 175 |
| 77.2 | 7788 | 1765 |
| 78.5 | 79032 | 1778 |
| 79.5 | 80102 | 179 |
| 80.6 | 8109 | 1801 |
| 81.6 | 8221 | 18134 |
| 82.8 | 8321 | 18244 |
| 83.7 | 8418 | 1835 |
| 84.6 | 851 | 1845 |
| 85.5 | 8584 | 1853 |
| 86.4 | 8689 | 18643 |
| 87.4 | 8789 | 1875 |
| 88.3 | 8883 | 1885 |
| 89.4 | 8968 | 1894 |
| 90.0 | 9044 | 1902 |
| 91.0 | 9168 | 1915 |
| 92.1 | 9264 | 1925 |
| 93.3 | 9406 | 19397 |
| 94.6 | 9516 | 1951 |
| 95.75 | 9653 | 1965 |
| 97.1 | 9792 | 19791 |
| 98.6 | 994 | 1994 |
| 100.0 | | |

# MONROE SIMPLIFIED METHOD FOR EXTRACTING SQUARE ROOT

The extraction of square roots by the Monroe method is a simple process of division and therefore is easy and rapid. It makes use of a table of factors which give accuracy to five significant figures in the root with an error of less than 5 in the sixth figure. The same method can be used for carrying the root to ten digits.

## The Monroe Method

To find $\sqrt{n}$ first determine N′ as follows:

For n between 1 and 100 inclusive, take n = N′

For n less than 1 or greater than 100, move the decimal point to the right or left in steps of two digits to arrive at N′ between 1 and 100.

Find the two consecutive values in the N′ column of the table between which N′ lies and select the values of A and D between the two selected N′ values.

## Instructions

*Step 1* Set n on the extreme left of the keyboard and enter as a dividend.

*Step 2* Set A, from the table, on the extreme left of the keyboard and add.

*Step 3* Set D, from the table, on the extreme left of the keyboard and divide.

The result in the upper dials of the Monroe, after decimal is pointed off, is the square root with an error of less than 5 in the sixth digit.

### Pointing Off Decimals in Roots

If n is greater than 1, start at the decimal point and working to the left, set off n into groups of two digits each. The number of such two digit groups to the left of the decimal point will be the number of digits to the left of the decimal point in the root. If the extreme left-hand group consists of only one figure, it should be counted as though a complete group.

If n is less than 1, start at the decimal point and working to the right set off the zeros preceding the first significant figure into groups of two zeros each. The number of such groups will be the number of zeros that should follow the decimal point and precede the first significant figure in the root. If the last right-hand group consists of only one zero, it should ***not*** be counted as a group.

**Example 1** $\sqrt{6942.3214} = 83.321$

*Step 1* Move decimal in n to left, 69.423214, which is between 69.3 and 70.4 in N′ column of table. From table, A = 69889 and D = 1672.

*Step 2* Set 69423214 on the extreme left of the keyboard and enter in the extreme left of the lower dials as a dividend.

*Step 3* Set A, 69889 on extreme left of keyboard and add.

*Step 4* Set D, 1672 on extreme left of keyboard and divide.

*Result* Upper dials 833207 or 83321

Inserting the decimal point gives the root, 83.321. The decimal point in the root is determined

by setting off the whole number 6942 into groups of two digits each, 69′42. Since there are two groups, there are, according to the rule, two whole number digits in the root, thus 83.321.

**Example 2** $\sqrt{0.000003912} = 0.0019779$

*Step 1* Move decimal in n to right, 3.912. From table, A = 3863 and D = 3931.

*Step 2* Set 3912 on extreme left of the keyboard and enter in the lower dials as a dividend.

*Step 3* Set 3863 on extreme left of the keyboard and add.

*Step 4* Set 3931 on extreme left of the keyboard and divide.

*Result* Upper dials 1977868 or 19779

Inserting the decimal point gives the root 0.0019779. The decimal point is determined by counting the number of full pairs of zeros to the immediate right of the decimal point in 0.00′00′03912. Since there are two such pairs of zeros (disregard the fifth zero) two zeros should follow the decimal point and precede the first significant figure of the root, thus 0.0019779.

**Example 3** $\sqrt{207.08425}$ to ten significant figures = 14.39042217

*Step 1* Move decimal in n to left, 2.0708425. From table, A = 2042 and D = 2858.

*Step 2* Set 20708425 on extreme left of keyboard and enter as a dividend.

*Step 3* Set 2042 on extreme left of keyboard and add.

*Step 4* Set 2858 on extreme left of keyboard and divide.

*Result* Upper dials 143906 or 14.391, root to five significant figures.

To carry out to ten significant figures, continue as follows:

*Step 5* Clear upper and lower dials. Disregarding decimal, set radicand 20708425 on extreme left of keyboard and enter as dividend.

*Step 6* Set 14391, root to five places without decimal, on extreme left of keyboard and divide.

*Result* Upper dials 1438984434

*Step 7* Average 1438984434 with the first approximation, 14391, by adding 1438984434 and 14391 with the left-hand digits aligned to obtain 2878084434 and divide that figure by 2.

*Result* Upper dials 1439042217, when pointed off, 14.39042217, root with maximum possible error of 1 in the tenth place

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Besides this method for square root, the booklet "Monroe Simplified Method for Extracting Roots" gives instructions for the extraction of cube and higher roots. It is Monroe Form 1191-S which will be furnished gratis by any Branch Office or by the company's Publications Department at Orange, N. J.

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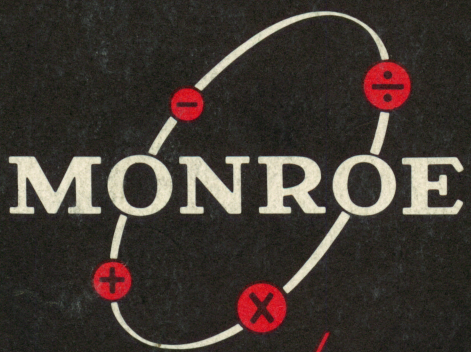


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